PATTERNS OF AERIAL TRIANGULATION BY INDEPENDENT MODELS

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ABSTRACT

The position of the non-zero elements in the coefficient matrix of the reduced normal equations are directly defined based on the existence of tie points between the models. The shape and the size of the matrix and the fill-in elements during its factorization depend on the geometric configuration of the models in the network, as well as the order of sequencing the models in the block. The ordering scheme of the models of homogeneous networks is investigated for variable geometrical parameters. With the aid of computer graphics, the pattern and size of the reduced matrix could be established from the topology of the network before any measurements take place. Manipulation of the ordering of models, beside other considerations of the method and strategy of the matrix decomposition, and the computation facilities would lead to the most efficient use of the computer storage and economy in computation time.

KEY WORDS: Aerotriangulation, Analytical, Computer Graphics.

1. INTRODUCTION

In aerial triangulation projects the computation and the decomposition of the matrix of normal equations play a decisive role in the costing and execution time of the project. The size of this matrix for large networks is usually of large order. However, this size can eventually be reduced to involve either the models' transformation parameters only or the ground coordinates of the tie points only. This procedure is well established and explained in many literature, e.g. Wong (1980). The number of models' transformation parameters is generally less than the number of the ground coordinates of tie points as unknowns in a block. Therefore, the reduced normal equations of the models' transformation parameters M is the one which is oftenly formed for economic computations and the one which is considered in this paper. Furthermore M is sparse, symmetric and positive definite. Therefore, advantage should be taken of these properties in order to reduce both storage requirements and computation time.

The shape and the size of the reduced matrix M, particularly the non-zero entries, depend on a number of parameters. The configuration of the block, the number of the strips s and their direction w.r.t. the block, the number of photographs in each strip g, the percentages of forelaps p and side laps q and the ordering scheme of the models are the main parameters. With the development and spread of microcomputers, the computation of large triangulation networks using such device became an objective (Julia, 1984; Klein, 1988). This necessitates the intorduction of economical storage schemes to make most efficient use of the limited core and memory capacity. The use of perephiral storage might be employed to overcome the problem (Lucas, 84). An attempt was made (Lucas, 84) to identify the structure and the pattern of the banded matrix M for p=q=67%, while the system was proposed to be solved by recursive partitioning. To avoid arithmetic operations with zero and storage of zeros within the regular band structure of M a nested dissection ordering technique is recommended (Stark and Steidler, 79). The same ordering technique is proposed by Shan (1988) for combined networks. Another proposal to arrange the sequence of unknowns to reduce the fill-in of new elements during factorization of M was to use the graph theory (Kruck, 84). A search routine had been deviced (Julia', 86) to identify which point connect which models, and how many models share a particular point.

This study is an attempt to investigate the possibility of automatic definition of the shape and size of M from the input data, and to establish the number and location of the non-zero elements m of M for variable parameters. The defined pattern of M gives in addition an insight to the required computing device and facilities, which form an important aspect in project planning. The conditions for particular ordering to achieve the minimum band width of M, or the least fill-in during its factorization to be established.

2. BASIC CONCEPTS

By virtue of its symmetry the matrix M would be presented by its main diagonal and the upper triangle only. A model which contributes to the structure of M should be joined at least to another model. This contribution is summarised in the following:-

+ Any model I(i) of order i contributes to M a matrix m(i,i) on its main diagonal as can be seen in figure 1. m(i,i) is conventionally called basic variance matrix. It is a 7x7 symmetric positive definite matrix, which is sparse on its own.

+ Any model I(i) which is joined with another model I(j) by one tie point, or more, contributes to M an off-diagonal matrix m(i,j) (figure 1), which is conventionally called basic covariance matrix. It is a 7x7 non-symmetric sparse matrix.

However, a number of models which are joined in a sequential order would form a line of models L(K).

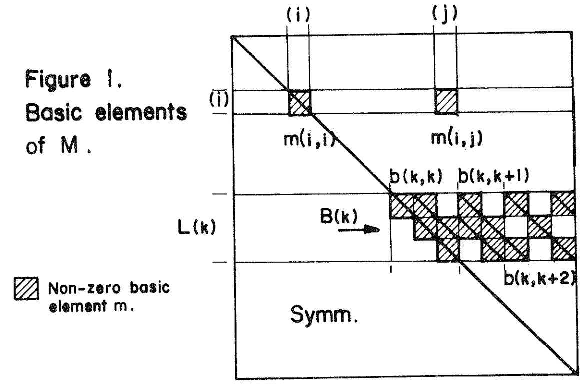


Figure 1. Basic elements of M.

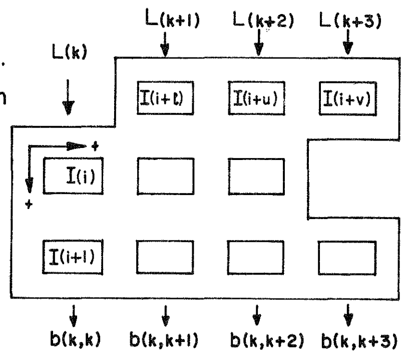


Figure 2·1· Correlation window.

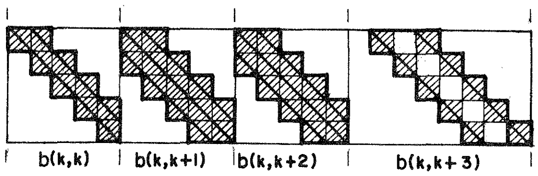


Figure 2·2. Matrix sub-block B(k)

The line L(k) would contribute to M the summation of the contributions of its models. A correlation window (figure 2.1) could be introduced to indicate each model I(i) and the joined (correlated) together with it models I(j) in the same line L(k) as well as in other lines L(k+t) in the block, such that j>i. The sum of the contributions of the models I(i) in one line L(k) shall constitute a sub-block B(k), which is a partitioned slice of M (figure 2.2). The sub-block B(k) usually consists of a "train of" sub-matrices b(k,k) on the main diagonal followed by at least one b(k, k+1) (figure 2.2).

The first "wagon" b(k,k) is a square matrix with its main diagonal as part of the main diagonal of M, and formed by the basic matrices m(i,i). The dimensions of b(k,k) corresponds to the range (No. of models) of L(k). It also has one off diagonal, or more, whose elements are the basic matrices m(i,j). The number of the off diagonals equals to the number of the correlated models with I(i) in the same line L(k) in the correlation window. The number of the subsequent wagons b(k,k+t) is defined by the number of the correlated lines L(k+t) in the correlation window. The number of columns of any of the submatrices b(k,k+t), which is generally rectangular, equals to the number of models in the line L(k+t) in the network. The number of the diagonals containing non-zero elements in the b(k,k+t) corresponds to the number of models in the line L(k+t) in the correlation window. The non-zero elements on these diagonals are formed by different arrangements of m(i,j). The intergration of the sub-blocks B(k) (for homogeneous networks) and/or the m(i,i) and m(i,j) (for irregular networks) would form the final pattern of M.

3. HOMOGENEOUS NETWORKS

By homogeneous networks is meant a network of regular geometry, i.e. there exist a similarity between the models in a strip, and repeatability between corresponding models in different strips. In such networks each and every model could be addressed by one and the same formula. Homogeneous networks perhaps better illustrate the application of the basic concepts to form the patterns of the matrix M. In addition it had been found much easier for these networks to construct the pattern of M from the sub-blocks B(k) rather than from individual rows of m(i,i) m(i,j).

4. STUDY CASES

The assigned values to p are 60%, 80%; to q- 20%, 60%, 80%. Different strategies are considered for the ordering of the models. The properties of the sub-blocks B(k) and their components b(k,k), b(k,k+t) as well as the resulting patterns of M are presented. For each case the non-zero elements m of M are indicated. The number and location of the fill-in elements which arise during the solution (Gauss elimination) are given. Also the most economic way of ordering the models in each case is investigated. The criterion for the economy is the ordering which would give the least number of fill-in elements during the solution.

4.1 p = 60%, q = 20%

Figure 3.1 represents the configuration of the photographs and the tie points in the block. The arrangement of the resulting models is shown in figure 3.2. For this case two ways of ordering the models are considered: Down-strip direction (→G), across-strip (↓S). The correlation windows for an arbitrary model I(i) in a line L(k) according to the ordering scheme are illustrated by figure 3.4. The lines I(1), I(s), I(g-1) indicate the boundary lines of these windows for models number (1),(s), (g-1) respectively in the line L(k). Apart from the first and last models in L(k), the correlation window remain the same for all the rest of models.

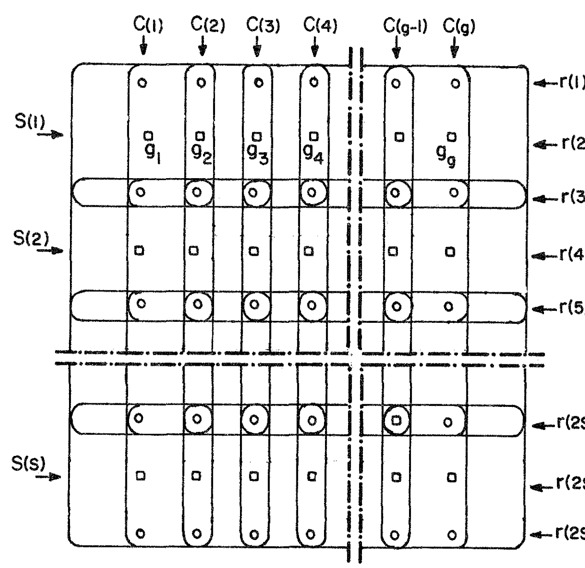


Figure 3·1· Set up of photographs and tie points, p=60%, q=20%.

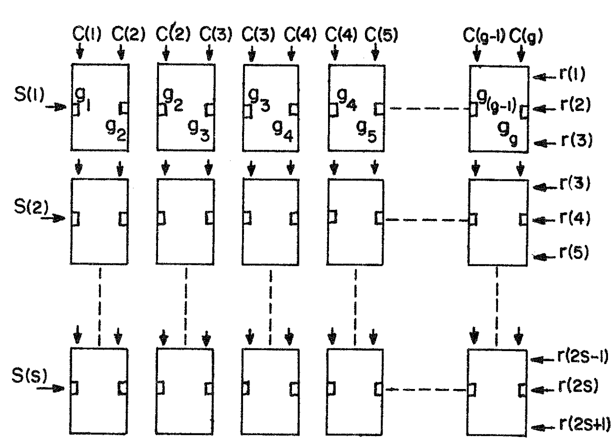


Figure 3·2· Arrangement of models, p=60%, q=20%

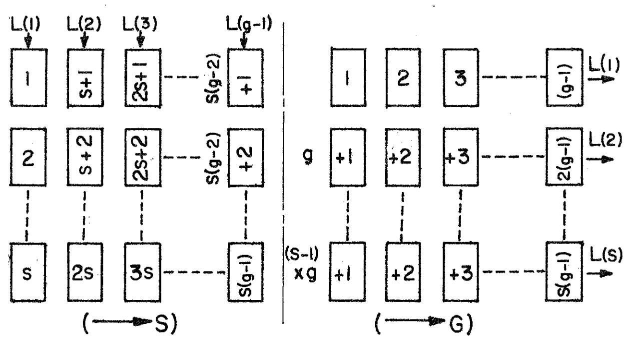


Figure 3·3· Ordering of models, p=60%, q=20%.

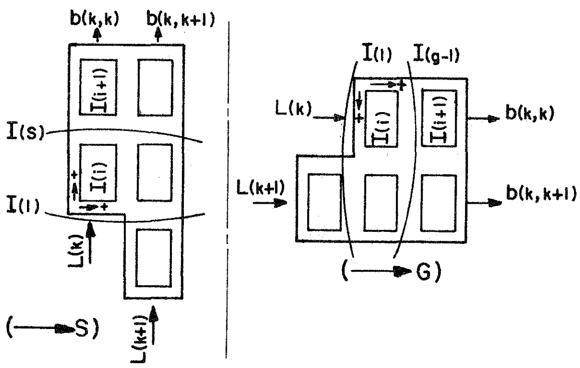


Figure 3·4· Correlation windows, p=60%, q=20%.

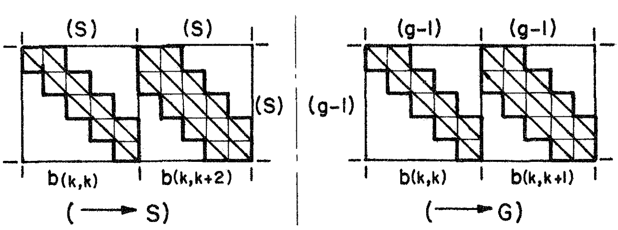


Figure 3·5· Matrix sub-block B(k), p=60%, q=20%.

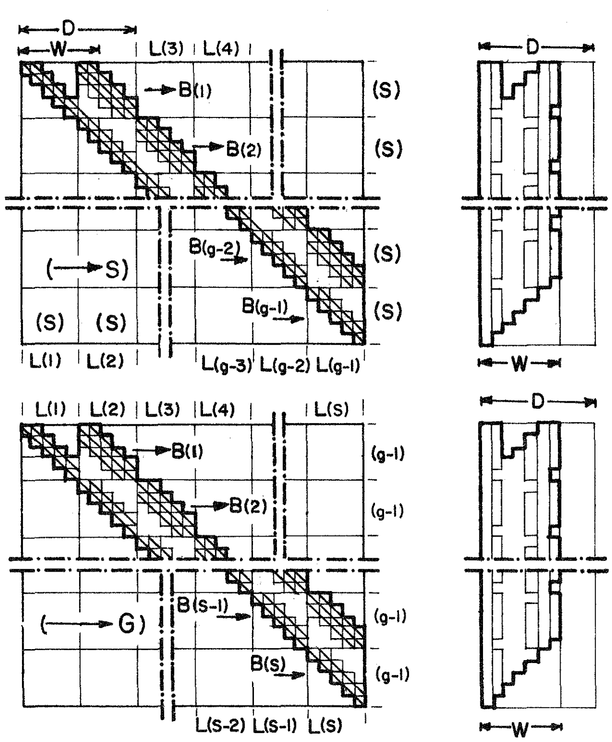


Figure 3·6· Patterns of M, p=60%, q=20%.

4.1.1 <u>Ordering (→G)</u>: The matrix sub-block B(k) consists of two component sub-matrices b(k,k), b(k, k+1) which are square and each is of size (g-1). b(k,k) has its main diagonal and a subsequent one off-diagonal as full of non-zero basic matrices m. b(k,k+1) has its main diagonal and one off-diagonal on each side as full. There are s sub-blocks B(k) forming the final pattern of M whose number of rows (or columns) becomes s(g-1) in terms of m.

No. of original basic matrices = 5sg-8s-3g+5
No. of fill-in (F.I.) basic matrices = (s-1)(g-2)(g-3)

4.1.2 <u>Ordering (→S)</u>: The matrix sub-block B(k) is formed of two s x s square sub-matrices b(k,k), b(k,k+1). b(k,k) has one off-diagonal beside its main diagonal, while b(k,k+1) is a tridiagonal matrix. The matrix M is constituted from (g-1) sub-blocks B(k) and has same size as in 4.1.1. The number of the original non-zero basic elements m should be the same as in 4.1.1.

No. of F.I. = (g-2)(s-1)(s-2).

It is noted that the numbers of F.I. elements are proportional to g^2, s^2 according to the ordering. The No. of F.I. in both cases is the same if s=g-1. Therefore, the economic No. of F.I is achieved by ordering in the direction of least number of models. The resulting patterns of M and its banded form are demonstrated in figure 3.6. It is also noted that whether the ordering is (→G) or (→S) the same pattern is achieved. The only differences are in the dimensions of b and the numbers of B.

4.2 <u>p = q=60%</u>

In some projects need might arise to increase the percentage of the fore-lap (p>60%) and/or the side lap (q>20%). Figure 4 represents the case of p=q=60% in a similar manner to figure 3. The condition for economic ordering is given in table 1.

4.3 <u>p=q=60% (+C)</u>

When the side lap is increased to 60% or more, full models between adjacent photographs in subsequent strips might also be formed, provided that proper alignment between these photographs do exist. Such models shall be called cross models and, if constructed and included in the computations, the case shall be denoted by (+C).

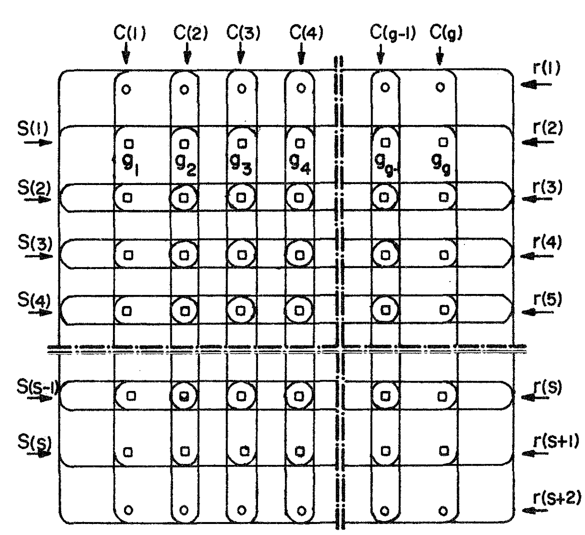


Figure 4.1. Configuration of photographs & tie points.

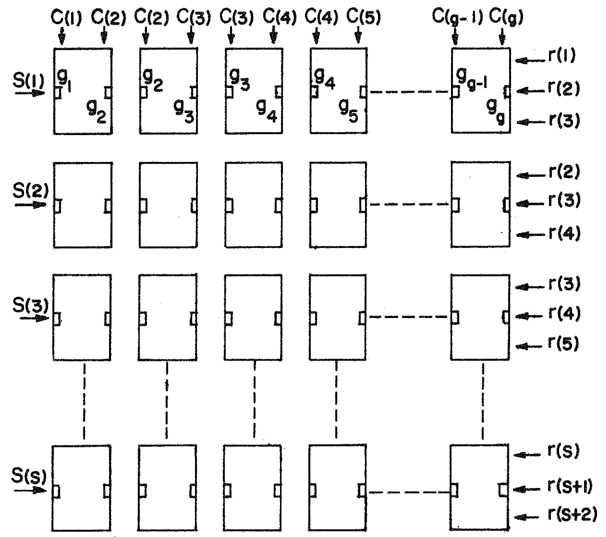


Figure 4.2. Arrangements of models and tie points.

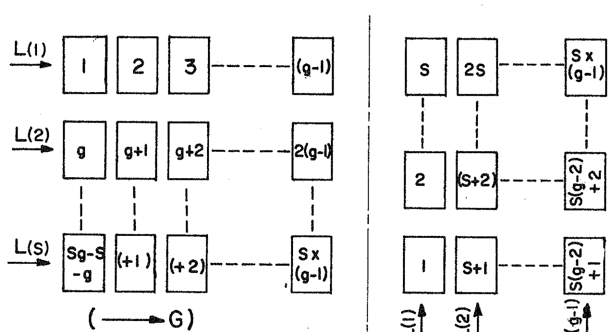


Figure 4.3. Ordering of models.

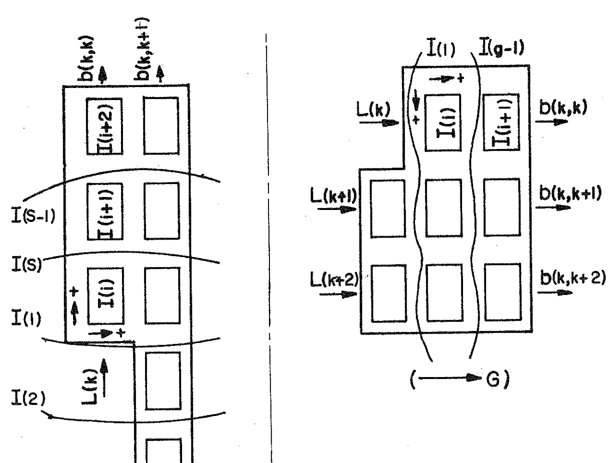


Figure 4.4. Correlation windows.

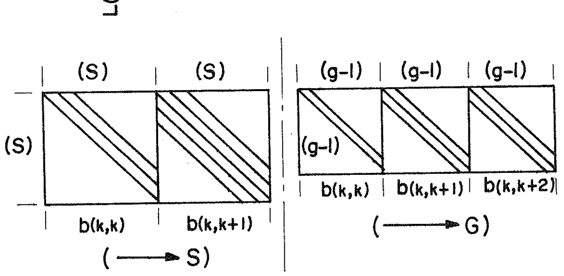


Figure 4.5. Matrix sub-block B(k)

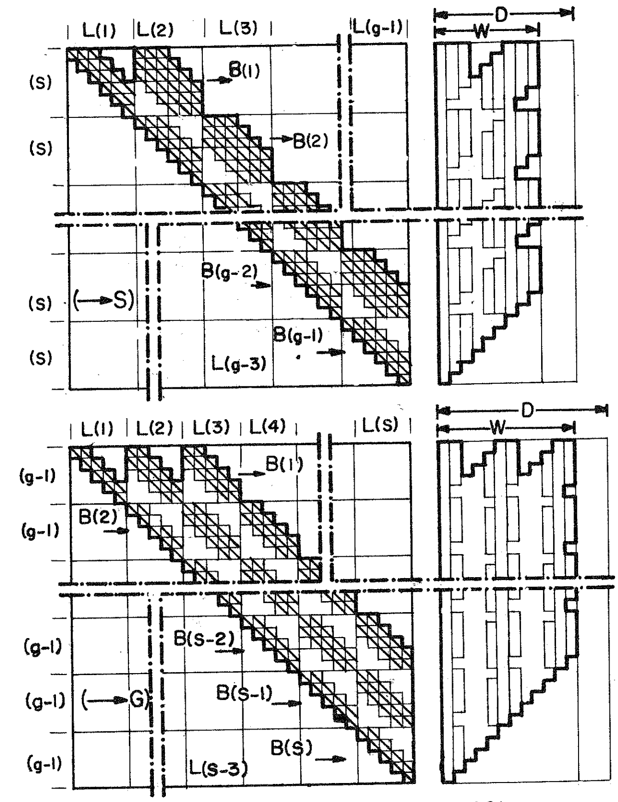


Figure 4.6. Matrix patterns, p=q=60%.

The ordering of models in this case was tried using different strategies, namely: (→G), (→S), diagonal front, Cuthill-Mckee method for minimum bandwidth (Cuthill, 1972) and spiral front. The ordering according to these strategies is demonstrated in figure 5 for a network of g=5 by s=3. The minimum number of F.I. as well as the minimum bandwidth had been achieved by ordering the models either (→G) or (→S) pending the dimension of the network (see conditions for economic ordering in table 1). The patterns according to these orderings are presented in figure 6. It should be noted that the resulting patterns are the same and the differences are only in the dimensions of b and the number of B constituting M.

4.4. p=80%, q=20%

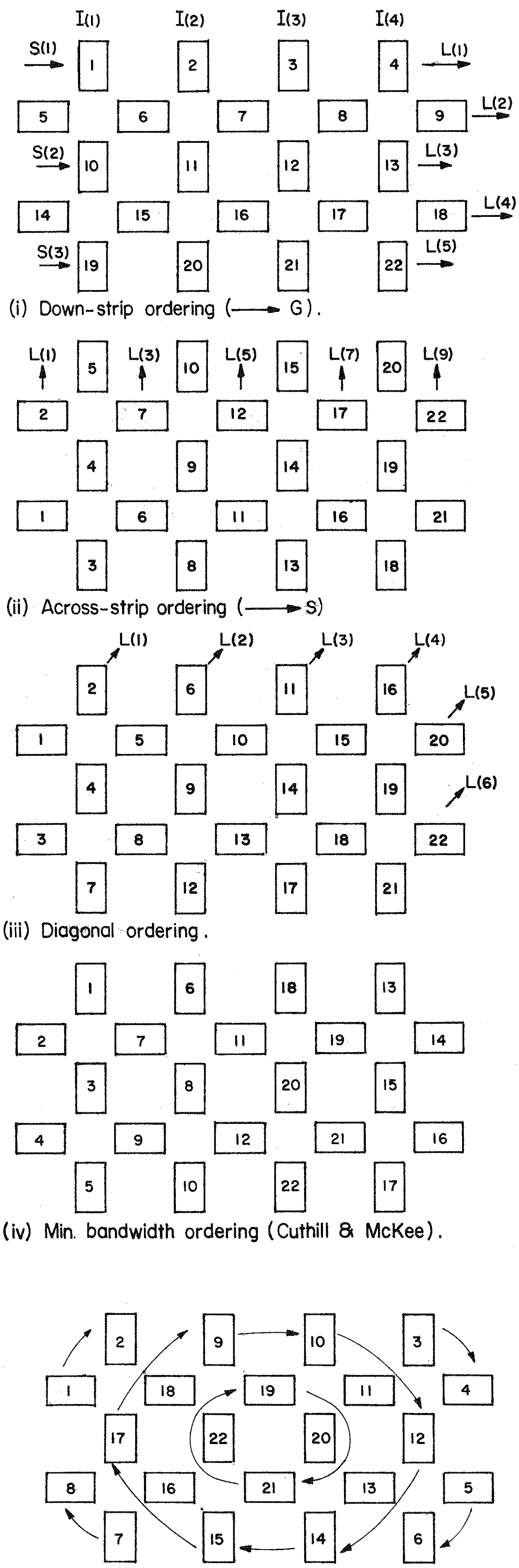
If the fore-lap is increased to 80% the models from consequetive photographs in a strip would be constructed with a base = 20%. This base would give intersections in the model space of less reliability if compared with the intersections produced from a base = 40% (figure 7·1). Therefore it might be advantageous in this case to construct the models in a strip from every other photograph (figure 7·2). Moreover this approach results in half the number of successively constructed models from a strip, which leads to economy in both observations and computations. It shall be assumed, therefore, for all the cases of p=80% that models from a strip are constructed from every other photograph. Figure 8 represents the patterns for ordering (→G) and (→S).

4.5. p=80%, q=60%

The patterns corresponding to this case are illustrated in figure 9.

4.6. p=80%, q=60% (+C)

In this case the cross models are introduced. The



(i) Down-strip ordering (⟶ G).

(ii) Across-strip ordering (⟶ S)

(iii) Diagonal ordering.

(iv) Min. bandwidth ordering (Cuthill & McKee).

(v) Spiral ordering front.

Figure 5. Various ordering strategies, p = q = 60% (+C).

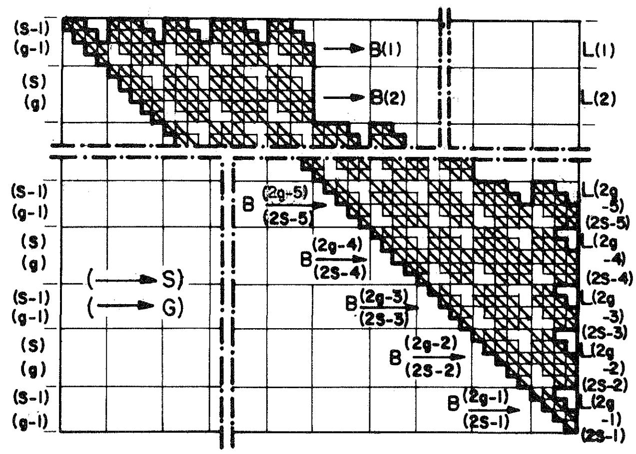


Figure 6. Pattern of M, p = q = 60%, (+C)

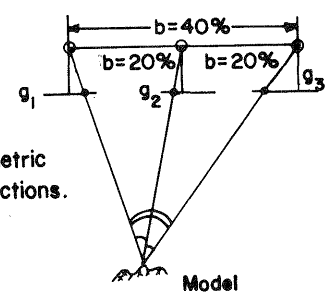


Figure 7·1· Photogrammetric rays intersections.

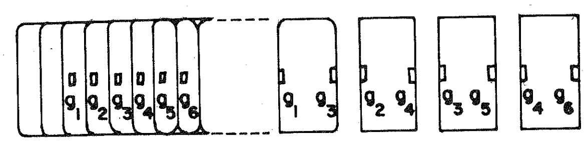


Figure 7·2· Strip of photographs (p = 80%) and models.

ordering (→S) is considered according to the two schemes denoted by (→S1), (→S2). The corresponding patterns are shown in figure 10.

4.7. p=q=80%

For this case the cross models would not be considered as they excessively increase the number of models. The patterns are presented in figure 11.

Table 1 summarises some information for the different cases.

5. NUMERICAL EXAMPLE

A land 6 x 20 kms is assumed to be covered by 23 x 23cm aerial photographs of scale 1:10,000. Lines of flights are assumed once to run parallel to the width, second - parallel to the length. These flight directions did exclusively satisfy in this example the economic conditions for the ordering (→G) and (→S) respectively. Table 2 gives a summary of the numerical values of the calculated parameters.

6. COMPUTER GRAPHICS

An attempt was done to produce the pattern and size of the matrix M by using Amstrad PC and Lotus graphics programme for a network s=3, g=5, p=60%, q=20%. The results are shown in figure 12. The numbers 1 simulate the basic matrix m, while the o represents the fill-in element. The non-zero envelop is also demonstrated by bold lines.

Table 1. Matrix Dimensions

p%	q%	T[1]	m[2]	Across-Strip Ordering (→S)			Down-Strip Ordering (→G)		
				D[3]	W[4]	F.I.[5]	D	W	F.I.
60	20%	s(g-1)	5sg-8s -3g+5	2(s)	s+2	(g-2)(s-1)(s-2) s≦(g-1)	2(g-1)	g+1	(s-1)(g-2)(g-3) s≧(g-1)
60	60	s(g-1)	8sg-13s -9g+15	2(s)	s+3	(g-2)(s-2)(s-3) $s \lesssim^{(6)}$ 2(g-1.3)	3(g-1)	2g	(2s-3)(g-2)(g-3) $s \gtrsim$ 2(g-1.3)
60	60 (+C)	2sg- (s+g)	32sg-46s -46g+66	5s-3	4s	(7g-11)(s-2)(s-3) s≦g	5g-3	4g	(7s-11)(g-2)(g-3) s≧g
80	20	s(g-2)	8sg-25s -5g+16	3(s)	2s+2	(2g-7)(s-1)(s-2) s≦0.5(g-1.5)	2(g-2)	g+1	(s-1)(g-4)(g-5) s≧0.5(g-1.5)
80	60 (+C)	2s(g-1) -g	55sg-151s -80g+218	5(2s-1) (→S1)	8s-3	(10g-22)(s-2)(s-3) +(g-5)(4s²-13s+13) $s \lesssim$ 0.5g; $g \gtrsim$ 7+(25/s)	5g-6	4g-1	(7s-11)(g-4)(g-5) $s \gtrsim$ 0.5g
80	60	s(g-2)	13sg-41s -15g+48	3(s)	2s+3	(2g-7)(s-2)(s-3) s≦(g-2)	3(g-2)	2g-1	(2s-3)(g-4)(g-5) s≧(g-2)
80	80	s(g-2)	23sg-73s -50g+160	3(s)	2s+5	(2g-7)(s-4)(s-5) $s \lesssim$ (2g-5)	5(g-2)	4g-5	(4s-10)(g-4)(g-5) $s \gtrsim$ (2g-5)

(1) Dimension of M in terms of basic matrix m. (2) No. of basic matrices. (3) Width of band in terms of m, taking b as a unit. (4) Width of band in terms of m, taking m as a unit. (5) No. of fill-in in terms of m. (6) $\gtrsim$, $\lesssim$: approximate relationship.

Table 2. Numerical Example

p%	q%	Ord. (→S) s / g	m / F.I.	Σ = m+F.I.	TW (+%)*	TD (+%)	Ord. (→G) s / g	m / F.I.	Σ	TW (+%)	TD (+%)
60	20	4 / 24	381 / 132	513	552 (7.6)	736 (43.5)	12 / 8	365 / 330	695	756 (8.8)	1176 (69.2)
60	60	6 / 24	873 / 264	1137	1424 (25.2)	1656 (45.6)	22 / 8	1065 / 1230	2295	2688 (17.1)	3528 (53.7)
60	60 (+C)	6 / 24	3294 / 1884	5178	6192 (10.3)	6966 (40.0)	22 / 8	4318 / 4290	8608	10304 (19.7)	11914 (38.4)
80	20	4 / 44	1104 / 486	1590	1680 (5.7)	2016 (26.8)	12 / 13	899 / 792	1691	1848 (9.3)	2904 (71.7)
80	60	6 / 44	2534 / 972	3506	3780 (7.8)	4536 (29.4)	22 / 13	2669 / 2952	5621	6050 (7.6)	7986 (42.1)
80	60 (+C)	6 / 44	10312 / 8097 / 8928	18409 / 19240	21240 (→S1) (15.4) / 21712 (→S2) (12.8)	25960 (4.10) / 25960 (34.9)	22 / 13	11586 / 10296	21882	27387 (25.2)	31683 (44.8)
80	80	11 / 44	8289 / 3402	11691	12474 (6.7)	15246 (30.4)	44 / 13	9454 / 11952	21406	22748 (6.3)	26620 (24.4)

*% increase over Σ.

7. IRREGULAR NETWORKS

Irregularity in networks could take place due to deviation of some flight parameters from the designed ideal, or some irregularity in the boundary of the photographed object.

7.1 Shift of models in adjacent strips

This is a very common phenomenon in aerial triangulation. It happens due to the inability during the flight to adjust the position of one photograph, or more, in a strip to exactly match the position of corresponding photograph in an adjacent strip (figure 13). The result is a shift in the area of the triple lap between two adjacent strips. In this case the identification of common tie points between strips to fall simultaneously in these areas becomes either difficult or impossible. This would lead to an intermediate model in one strip being joined to only two models, and not three, in the adjacent strip (assuming q=20%,p=60%). Thus the subblock B(k) of the matrix M would take one of the patterns illustrated in figure 14. Here one off-diagonal (either the upper or the lower) of the submatrix b(k,k+1) becomes zero.

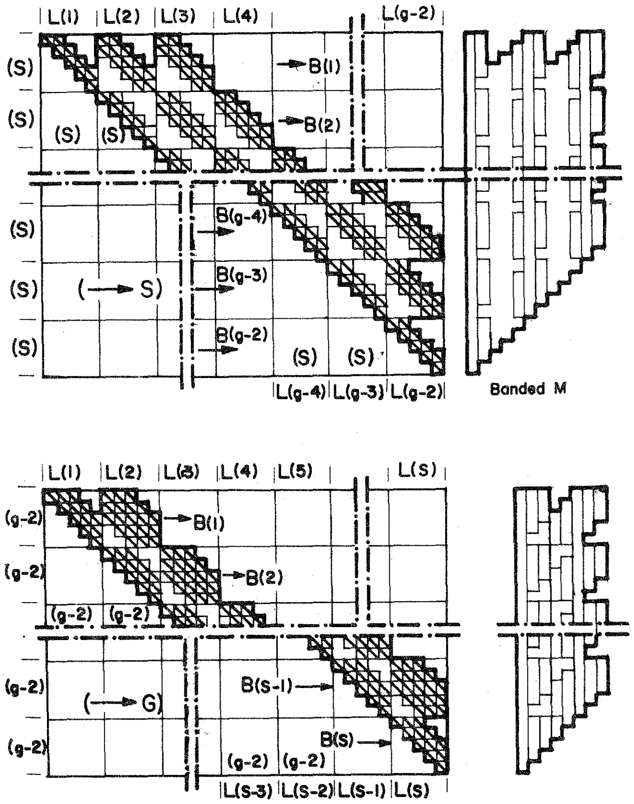


Figure 8. Patterns of M, p=80%, q=20%.

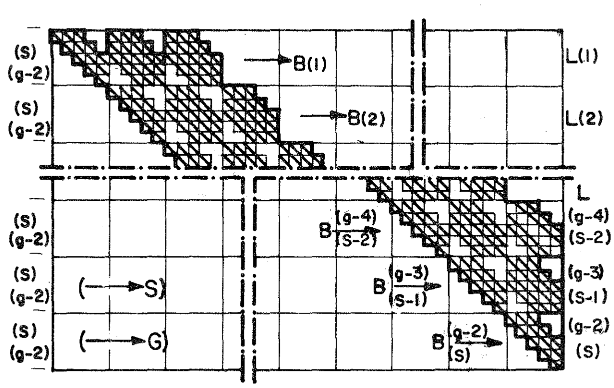


Figure 9. Patterns of M, p=80%, q=60%.

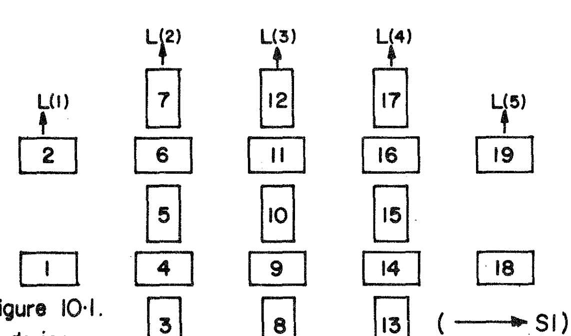


Figure 10·1. Ordering.

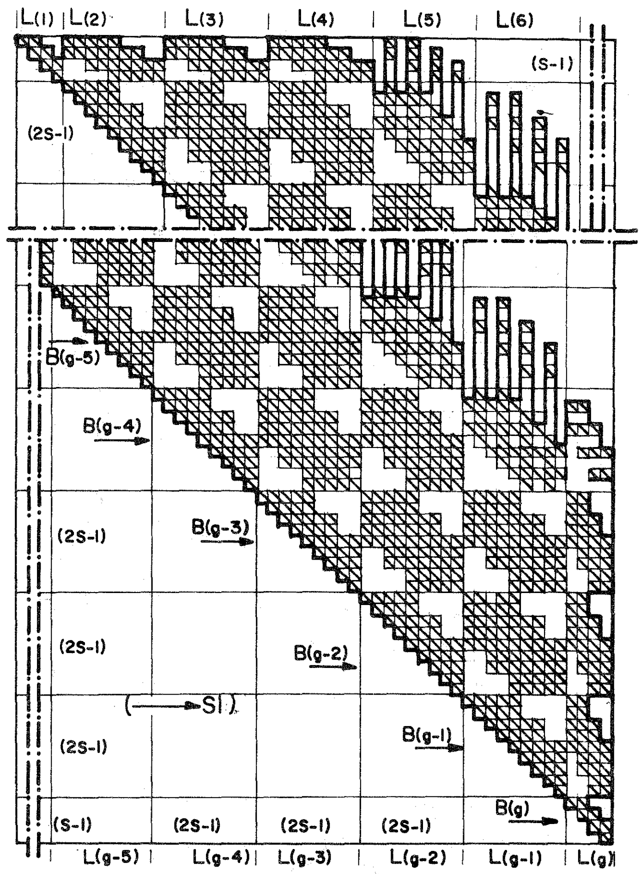


Figure 10·2. Pattern of M, p=80%, q=60%, (+C).

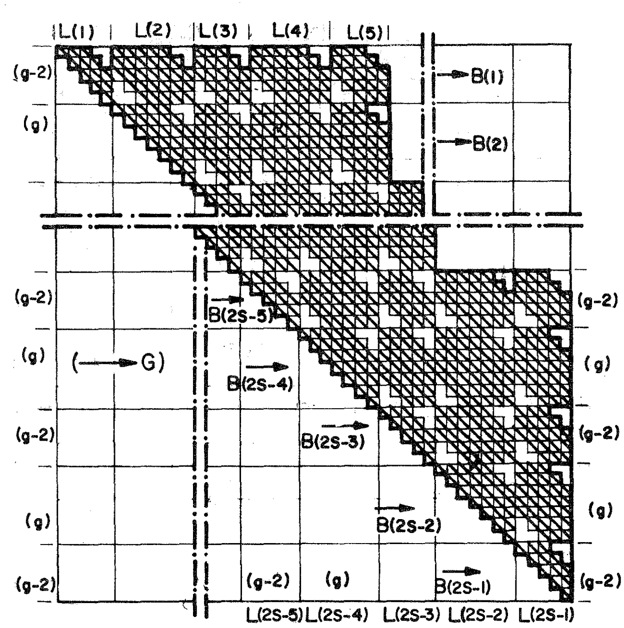


Figure 10·3. Pattern of M, p=80%, q=60%, (+C).

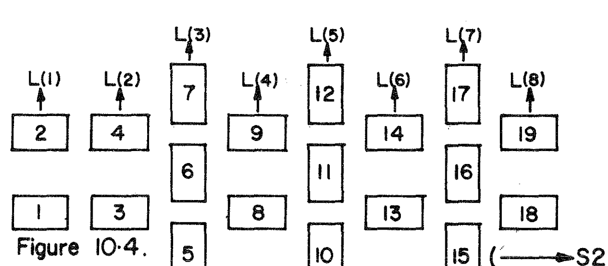


Figure 10·4. Ordering.

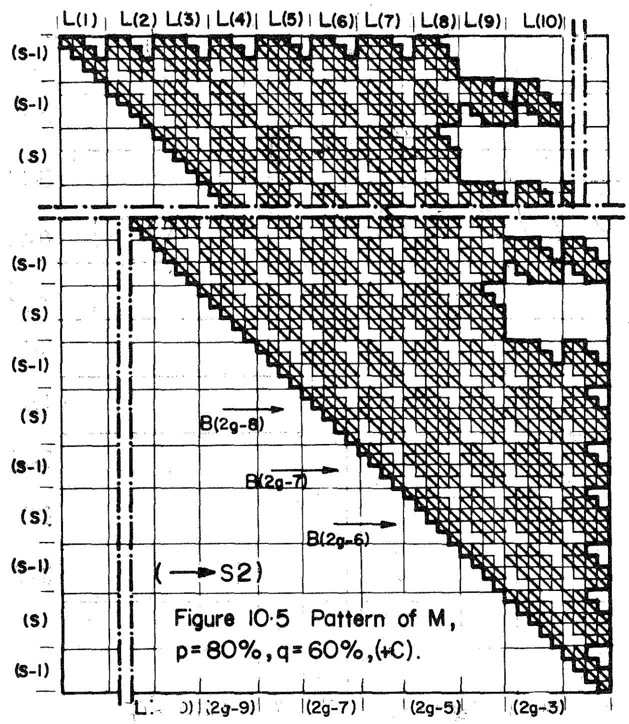


Figure 10·5 Pattern of M, p = 80%, q = 60%, (+C).

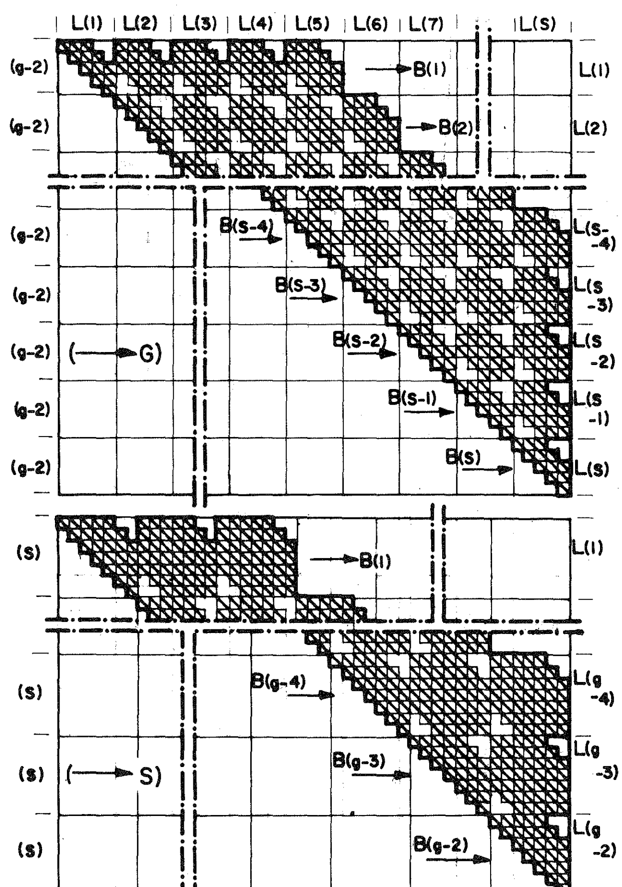


Figure 11. Patterns of M, p = q = 80% .

If the side lap in this case is increased to 60%, it would not be possible to construct the cross models, as they become incomplete. The same principles described for homogeneous networks could be extended to cases of increased p%, q%.

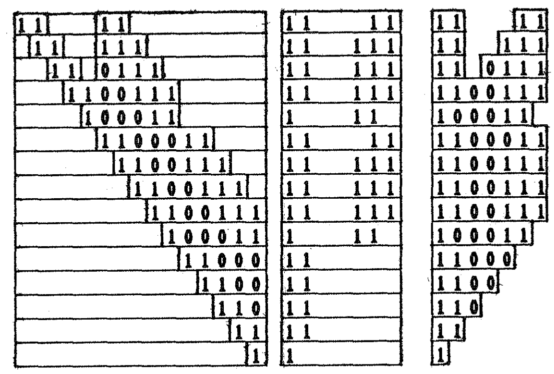
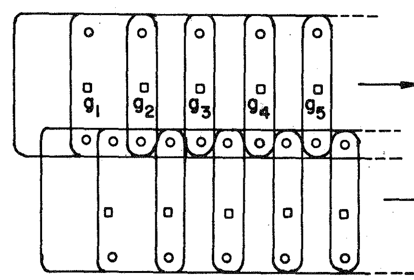


Figure 12. Computer output of pattern of M, p=60%, q=20%, g=5, s=3 , m ≡ 1, F.I. ≡ 0.



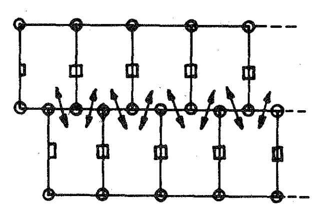


Figure 13. Shift of strips: arrangement of photographs, tie points and models, p=60%, q=20%.

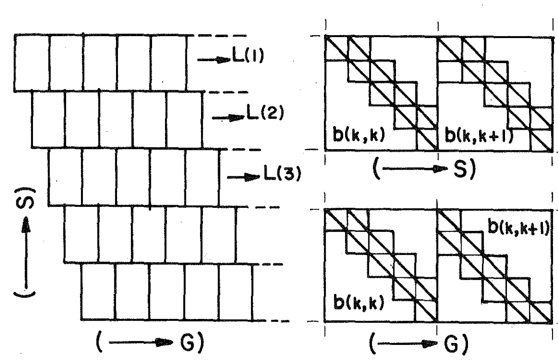


Figure 14. Sub-block B(k) for shifted strips.

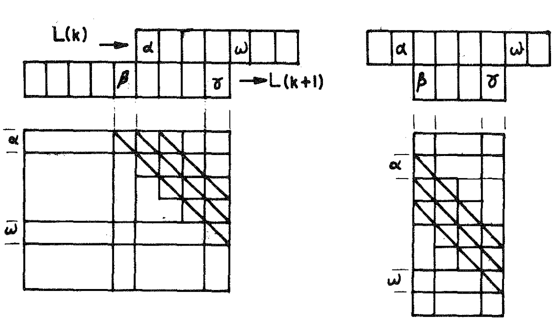


Figure 15. Sub-matrix b(k, k+1) for irregular boundaries.

7.2 Irregular boundary

The irregular boundary of the photographed area, or the existence of lakes or large water bonds, or the intentional extension of one strip, or more, to cover a ground control point outside the boundary, or any other reason might give rise to a situation whereby the numbers of models in adjacent strips are not the same, and/or the starting models in them might not coincide. The change in M would take place in the structure of the correlation submatrix b(k, k+1). The key to define this structure is to find the order of the first and last models (α,ω) in a line L(k) and the order of the joined with them first and last models (β,γ) in the following line L(k+1); and the number of models joined with each. It should be noted that α and/or β is the first model in its line, also ω and/or γ is the last. The order of these models (α,β), (ω,γ) gives the start and end non-zero basic covariance matrix m of one diagonal (if they fall on one diagonal) or two boundary diagonals respectively (if they fall on different diagonals). The number and location of non-zero diagonals of b(k,k+1) are then identified as in the two illustrated cases by figure 15.

The resulting pattern of M for any combination of irregularities with different p%, q% could be constructed by integrating the appropriate basic concepts.

7.3 Irregular scale and orientation of photography

The irregular scale and/or orientation of photography could arise when different date photography are used for aerial triangulation. This might result in a model being connected with several other models by varying numbers of tie points. In this case a search routine should be employed (Julia, 86) to identify the points common to particular models, and which models are connected by one and the same point. The minimum bandwidth strategy for ordering the models might be suitable for this situation.

8. CONCLUSION

The established patterns of M and the numerical examples make it possible to conclude the following remarks and recommendations:
(1) The sparsity and structure of the coefficient matrix M has the property of regular band pattern, where the non-zero basic matrices m lie within a diagonal band W. The decomposition of M can be performed within this band.
(2) The storage of the matrix M is most suitably accomplished by diagonal storage. The storage space of the non-zero envelop is the most economical. This storage system is most suitable for solution by Gauss elimination.
(3) The storage of M with its full half bandwidth W would require extra storage facilities from 5%-25%.
(4) If the solution is sought by partitioning, the best candidate for a partitioned unit is the sub-matrix b. The half bandwidth of M in this case is D, with 25%-70% additional storage requirement.
(5) The ordering of the models has a prime influence on the size of M. The number of F.I. $\propto s^2, g^2$ for ordering across - strip, down-strip respectively.
(6) The conditions for economical ordering depend on p%,q%,s and g. These conditions could be set in the computer program to resequence the models. Together with a suitable computer graphics facility manipulation of the ordering for least size of M could be achieved.
(7) The rise in the p%, q% increases the number of models. The increase is almost linear with every 20% step increment of p & q (q=20% $\equiv$ q=40%).
(8) The inclusion of the corss models, if they are possible to be constructed, almost doubles the number of the constructed models and quadrable the size of M.
(9) The inclusion of the cross models is anticipated to strengthen the solution. The significance of the improvement yet to be established versus the cost of additional observations and increase in storage and computation time. In this case the economy in storage and computation of M composed of models' transofrmation parameters against coordinates of the points should be investigated.
(10) For very large M,perepherals are recommended to be used with micro computers to transfer to and from the core the active part of M necessary for forward reduction or back substitution of one step at a time.

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Errata to Table 1

Ordering (→S2) for p=80%, q=60%, (+C)

D	W	F.I.
5(2s-1) (→S2)	8s-2	(13g-37)(s-2)(s-3) +(g-6)s(2s-1) $s \leq 0.5g; g \leq 7+(25/s)$