Deformable Model for Image Segmentation

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ABSTRACT:

This paper presents an approach for image segmentation using Minimum Description Length (MDL) principle and integrating region growing, region boundary fitting, model selection in an integrated manner, on which the final result is a kind of compromise of various sources of knowledge such as the original grey level image and deformable object models. The deformable model is a closed polygon depicted by a number of straight line segments. The formulae for encoding the image intensity and shape of region are presented. The whole process is carried out by a split-and-merge mechanism which is based on a new data structure. We also introduce the method for optimal curve fitting. We believe that such approach can be used in the segmentation and feature detection for remote sensing image, urban scene image as well as in the automated integration of GIS, remote sensing and image processing.

KEYWORDS: Image Segmentation, Image Analysis, Feature Extraction.

1. INTRODUCTION

Image segmentation is one of the fundamental requirements in image analysis. The techniques for image segmentation roughly fall into two general processes:

1). Edge detection and line following. This category of techniques studies various of operators applied to raw images, which yield primitive edge elements, followed by a concatenating procedure to make a coherent one dimensional feature from many local edge elements.

2). Region-based methods. Region-based methods depend on pixel statistics over localized areas of the image. Regions of an image segmentation should be uniform and homogeneous with respect to some characteristics such as grey tone or texture. Region interiors should be simple and without many small holes. Adjacent regions of a segmentation should have significantly different values with respect to the characteristics on which they are uniform. Boundary of each segment should be simple, not ragged, and must be spatially accurate [Haralick and Shapiro,84].

Image segmentation is hard because there is generally no theory on it. Segmentation techniques are basically ad hoc and differ precisely in the way they emphasize one or more of the desired properties and in the way balance and compromise one desired property against another.

The whole content of this paper is concentrated on the region-based methods. Region-based image segmentation techniques can be classified as: measurement space guided spatial clustering, single linkage region schemes, hybrid linkage region growing schemes, centroid linkage region growing schemes, spatial clustering schemes, and split and merging schemes [Haralick and Shapiro] [Benie] [Besl,88a,88b] [Bharu] [Blanz] [Haddon] [Hu] [Liou] [Pappas] [Rodriquez] [Snyder] [Sumanaweera] [Tsikos].

Recently, There is increasing interest in applying the information theory to automatically interpret and analysis the image data [Foerstner and Pan] [Kim] [Hua,89a,89b,90] [Leclerc,89,90] [Leonardis] [Meer] [Pavlidis]. The fundamental concept in information theory is the idea that the amount of information derived from some event, or experiment, is related to the number of degrees of freedom available beforehand or the reduction in uncertainty about some other event gleaned from an observation of the outcome. Among the tremendous tools provided by information theory, Minimum Description Length (MDL) principle has been quite successfully applied in computer vision field.

MDL principle studies estimation based upon the principle of minimizing the total number of binary digits required to rewrite the observed data, when each observation is given with some precision. Instead of attempting at an absolutely shortest description, which would be futile, it looks for the optimum relative to a class of parametrically given distributions. This MDL principle turns out to degenerate to the more familiar Maximum Likehood (ML) principle in case the number of parameters in the models is fixed, so that the description length of the parameters themselves can be ignored. In another extreme case, where the parameters determine the data, it similarly degenerates to Jaynes's principle of maximum entropy. *The main power of the MDL principle is that it permits estimates of the entire model, its parameters, their number, and even the way* *the parameters appear in the model; i.e., the model structure* [Ressanen,78,83,84].

The MDL principle can be generally expressed as

$$L(x,\Theta) = L(x/\Theta) + L(\Theta)$$

where

$L(.)$ is a measure of the uncertainty of an event and its unit is "bit".

$L(x,\Theta)$ is the total number of bits to describe the observed data when we introduce the model. "x" expresses the observed data, and "Θ" represents the model parameters.

$L(x/\Theta)$ is the number of bits to describe the data if assuming the model is known.

$L(\Theta)$ is the number of bits to describe the model.

$L(x,\Theta)$ is the least information content required to remove the uncertainty in the observation and describe the model. Thus, the number of bits in a description required for the interpretation of the observation becomes a measure of simplicity.

There are two assumptions or requirements when applying MDL principle: the model type or class must be known beforehand (number of parameters may be unknown), and a priori language (or encoding scheme) which transfers the model into units of "bits" must be defined. In practice, we always have certain knowledge on the model, otherwise it is totally impossible to inference any thing from the observation. In a quite ideal situation where probability distribution depicting the behaviour of the model is known, a negative logarithm (usually in base 2) can be used to get the length description in "bits". In many cases, such probability distribution is difficult to achieve and the optimal encoding described by Shannon's first theorem is difficult to find, then only approximated optimal coding scheme (descriptive language) can be formulated. The resulted estimation is the relative optimal result constrained by the approximated encoding scheme. So MDL, used as the notion of simplicity, strongly depends on the choice of the descriptive language.

There are already a number of work using MDL principle for the image analysis. Darell [Darell90] formulated the segmentation task as a search for a set of descriptions which minimally encodes a scene. A new framework for cooperative robust estimation is used to estimate descriptions that locally provide the most saving in encoding an image. A modified Hopfield-Tank network finds the subset of these description which best describes an entire scene, accounting for occlusion and transparent overlap among individual descriptions.

Fua and Hanson [Fua,89a,89b,90] applied the MDL principle to extract building from the image. Their interpretation consists of two steps: 1). derive a set of likely hypothesis' of image description using search or estimation techniques. Here all available knowledge on the type of objects and on efficient strategies may be explored. 2). choose the best competing hypothesis based on the simplicity of the description. The simplicity or likehood is measured by the number of bits necessary to describe specific realization of the model and the deviation of the actual image from the ideal model. The problem of evaluating competing image descriptions is the incompatibility of photometric data (the original observations in digital image) and the complexity of the model.

Leclerc has done some elegant work on image segmentation using MDL principle. In 1989, he [Leclerc,89] presented a hierarchical optimization approach to the image partitioning problem: that of finding a complete and stable description of an image, in term of a specified descriptive language, that is simplest in the sense of shortest description length. The first stage in the hierarchy uses a low-order polynomial description of the intensity variation within each region and a chain-code-like description of the region boundaries. By using a regular-grid finite element representation for the image, the optimization technique, called a continuation method, reduces to a simple, local, parallel, and iterative algorithm that is ideally suited to the Connection Machine.

In his later work [Leclerc,90], he added an another layer (region grouping) which groups together regions that belong to a single surface. One possible basis is the "good continuation" of segments of region boundaries. A second possible basis is the "good continuation" of the intensity variation within the regions. The interpretation of good continuation means that the intensity variation within a group of regions is simpler to describe using a single polynomial model than with the independent polynomials (one per region) original recovered by the segmentation algorithm.

Another relevant work was done by Keeler [Keeler,91] who regarded a "segmentation" as a collection of parameters defining an image-valued stochastic process by separating topological (adjacency) and metric (shape) properties of the subdivision and intensity properties of each region. The priori selection is structured accordingly. The novel part of the representation, the subdivision topology, is assigned a priori by universal coding arguments, using the minimum description-length philosophy that the best segmentation allows the most efficient representation of visual data.

The work we present in this paper is in the same spirit of Leclerc's work. We employ the minimum Description Length (MDL) principle to find the simplest partition (segmentation) of images based on the generic models. Leclerc's idea of "region grouping" is based on the "good continuation" of segments of region boundaries and intensity variation within the regions. We consider that such "good continuation" of region boundaries is actually utilization of shape of generic model, e.g., closed polygon consisting a number of straight line segments, or circle, ellipse, etc. It is very difficult to imagine that we can use a general shape modelling for all kinds of images, in other words, the generic shape strongly depends on the application and on what we expect to extract from the images. The important feature of our approach which is different with Leclerc's is in integrating region growing, region boundary fitting, model selection in an integrated way, which is easy to be extended to incooperate more types of shape constrains as well as structural constraints.

2. MULTI-LEVEL REPRESENTATION FOR SEGMENTATION

The process of image analysis is often in the fashion of consisting a hierarchical representation where each level is associated with an objective with it. For the purpose of image segmentation, a three level representation scheme is used in our research as illustrated in Fig.1 (on the pages after "references"), which composes the original image, segmented image and vector data. Under this scheme, the segmentation is the result from the operations carried on segmented image and inter-reactions from the original image and vector data. Split-and-merge is the main mechanism in the segmentation procedure, which merges small regions into more meaningful big region, or split the big region into small regions when required. An initial segmentation is performed to get basic regions from the original image. After each level of split-and-merge, vectorization procedure transfers the region boundary into vector description, followed by a curve fitting algorithm which derives a compacted vector data based on the generic model. Based on the result of curve fitting, a measurement is calculated using MDL principle to describe the uniformity of region by shape constraints. Such measurement is integrated with the information derived from the original image intensity to improve the decision making of split-and-merge of regions.

The formulae for encoding the intensity and shape of regions is described in section 3, and section 4 and 5 introduce the region growing and curve fitting algorithm respectively.

3. USING MDL TO ENCODE THE INTENSITY AND SHAPE OF REGIONS

<u>Encoding intensity information of region</u>

We model the interior intensities of an image region by a smooth intensity with a Gaussian distribution of deviations from the surface. The formula for this problem has been solved by Hua and Hanson [Fua,90], is adopted in our work as formula (1):

$$L_I = n_1(\log\sigma + c) + 8n_2 + [n_1\log(\frac{n_0}{n_1}) + n_2\log(\frac{n_0}{n_2})] + N_p \qquad (1)$$

Where

L_I : number of bits to describe intensity information in a region

$$c = \frac{1}{2}\log 2\pi e$$

n_0 : total number of pixels in one region
n_1 : number of pixels in the Gaussian peak
n_2 : number of outliers
N_p : number of bits to describe surface model
σ : standard deviation of intensity noise

In (1), first item is the cost of Huffmann-encoding the pixels in a Gaussian peak, second item is the cost of encoding the pixel outliers, third one is the entropy for encoding the pixel on whether it is or is not a anomalous and fourth term specifies the coding of the model.

<u>Encoding shape of region</u>

Leclerc [Leclerc,89] has developed certain formula to encode the boundaries based on the assumption that each segment of the boundary is described in terms of straight line and the deviation from the line. The deviation is modelled as the perpendicular distance of the point to the line, which obeys a Gaussian distribution. We argue that while it is reasonable to describe the image intensity deviation by a Gaussian distribution because the quantization of image intensity holds a large range (usually from 0 to 255), it is not quite adequate to apply the same principle to encode the shape. A simply connected curve is one on which any point on the curve has at most two neighbours which lies on the curve. In this paper, we use the same modelling of shape as Leclerc's, i.e., the ideal region boundary composes a number of straight line segments. For a straight line, its digitalization fulfils the chord property which states [Hung,85] that "a digital arc A is said to have the chord property if for every two digital points, the chess-board distance of any point of arc A to the straight line nowhere exceeds 1". Based on such observation, we consider the points on a curve which do not meet the chord property as outliers, and such outliers are constrained by their neighbours. The method of determining the optimal positions of nodes connecting straight line segments is represented in section 5.

The cost required to describe the image region shape is formulated as:

$$L_S = 0 + 2m_2 + [m_1\log(\frac{m_0}{m_1}) + m_2\log(\frac{m_0}{m_2})] + \log(D_xD_y)m_S \qquad (2)$$

Where

L_S : number of bits describing the shape of a region
m_0 : total number of points on the boundary
m_S : number of straight line segments on the boundary
m_1 : number of points fulfilling the chord property

m_2 : number of outliers
D_x, D_y : number of pixels along x and y direction of the image

In accordance with (1), we also use 4 items encoding the shape of regions. For the points on the curve which meet the chord condition, no additional coding is needed as far as the nodes specifying the straight line segments are known, so first item in this case is zero. The second item in (2) is the number of bits describing the outliers. If the boundary is encoded in Freeman chain code, 3 bits is required to store each pixel (for 8 directions). But if we store the edges between the pixels instead of pixels themselves, only 2 bits is necessary (for 4 directions) (Such way of representing the boundary allows us to depict closed region, single line as well as point). The third term is in the same meaning as equation (1). The final component is used for the coding of nodes connecting straight line segments.

Finally, the total information required to describe a region considering intensity and shape is the sum of L_I and L_S, namely,

$$L = L_I + L_S \tag{3}$$

4. REGION GROWING ALGORITHM

The region growing algorithms in the literature can be classified into three categories: pure merging, pure splitting. and split-and-merge schemes. In the first scheme, the picture is first divided into many small primitive regions (even pixels) which are then merged to form larger regions on the basis of certain homogenous criterion. In contrast, a pure split method views the entire image as the initial segmentation. Then, it successively splits each current segment into if the segment is not homogenous enough. In split-and-merge scheme, the efficiency of processing is improved by first partitioning the image into square subregions and then splitting them further if they are not homogeneous. Afterward, a merging process is applied to those adjacent segments that satisfy some uniformity criterion. Pyramid quad structures is often employed as the basic data structure. A detailed implementation on split-and-merge scheme can be found in [Chen].

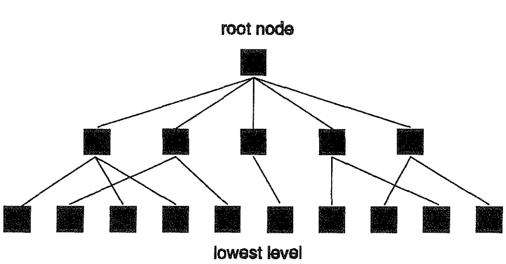

Fig.2

In order to implement our philosophy, quad tree structure is not quite valid because we consider that the splitting and merging in this data structure is usually not carried out on where the real object region boundary occurs. When we want to interactively include the shape as one important component of region uniformity, we have to find out a data structure which can describe each region more directly. Therefore, "N-node tree" has been developed in our current algorithm. An example of "N-node tree" is illustrated in Fig.2 .

One can view "N-node tree" as a generalization of quad tree data structure in the sense that while number of children under a node in quad tree is fixed to 4, the "N-node tree" decides the number of children according to the number of small regions from which the big region merges. In Fig.2, the lowest level of the tree is actually the original image pixels which is indexed from left to right, up to down on the image, and the top level or root node represents the whole image. The intermediate levels of the tree describes the segmentation results at different stages. One node on each level represents a complete region which has a unique and sequential label on this level. The original image and the segmented regions can be corresponded via tracing through the "N-node tree". In practice, a label image, which has the same size as the original image, is created and is valued by its corresponding label in order to facilitate the indexing.

The whole segmentation procedure is carried out in 4 phases:

1). forming lowest level of the tree. It is a simple indexing which puts original raster image pixel into the lowest level of the tree.

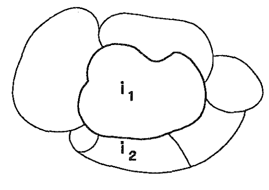

Fig.3

2). initial merging based on statistical test on image intensity. It is a traditional method used in split-and-merge algorithm, which examine the merging result with the previous small regions using statistical method to decide whether such merging is good or not. The decision making for merging is shown in Fig.3. For region i_1, in order to find the neighbour region to merge with, all the regions surrounding region i_1 should be examined and the one having most closed uniformity with region i_1 is selected as candidate region to merge with (say,e.g. region i_2). If uniformity measurement of region i_1 and i_2 is within certain threshold, these two regions are merged to form big region. This step can be regarded as providing hypothesis for possible homogeneous regions.

3). adaptive merging based on MDL, combining grey level and shape information. The decision making procedure for merging is similar as in last step. But in this case, a candidate region is choosed if it has the minimal length measure by formula (3) suppose it merges with region i_1 (we still use Fig.3 as example), compared with other regions surrounding region i_1. For selected candidate region i_2, it merges with region i_1 if the MDL measurement after merging is smaller than the sum of individual MDL measurements from two regions before the merging.

We can express such procedure more rigorously in mathematic formula as the following:

Denoting R^k_i as the i^{th} region at k-stage merging (each region under each level is uniquely and sequentially labelled)

for each region i_1 in k^{th} merging result,

$$R_j^{k+1} = R_{i_1}^k \cup R_{i_2}^k$$

$$if \;\; L(R_{i_1}^k \cup R_{i_2}^k) \leq L(R_{i_1}^k) + L(R_{i_2}^k) \qquad (4)$$

where

R^{k+1}_j is $(k+1)^{th}$ merging result from current regions i_1 and candidate region i_2. After each level of merging, labels are updated to produce a unique and sequential label for each region. So j has not necessarily same label value as i_1 in k^{th} level.

and,

candidate region i_2 is decided by

$$L(R_{i_2}^k \cup R_{i_1}^k) < L(R_l^k \cup R_{i_1}^k) \qquad (5)$$

for all l (i_2 and l is the neighbours of i_1).

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4). removing small abnormal regions. In this step, small abnormal regions are merged with an adjacent larger regions. The existence of small abnormal regions may be due to: a), there are small objects on the big object surfaces, and these small objects are out of our interest; b), the existence of high-frequency noises on the image. For this purpose, formula (5) is still used to find most suitable big region. Sometime, several small areas can congregate together, which make it impossible to find a direct neighbouring big region to merge with. Under this circumstance, algorithm should allow current small region to jump over neighbouring small regions to the nearest big region.

5. BOUNDARY FITTING ALGORITHM

Boundary fitting, or curve fitting belongs to the problem of shape analysis, which deals with using minimum number of points to represent a curve under the certain criterion for error because of data reduction. It is the core issue in the line generalization of Cartography. Our problem here is slightly different in the way that we have some assumptions: 1) the boundary for a region is closed; 2), object models are known, or in other words, some properties of shape for objects are a priori knowledge, among other assumptions if specified by the applications. Such assumptions or a priori knowledge will greatly reduce the complexity of curve fitting. In the following, we introduce our approach on optimal curve fitting of the region boundary under the assumption that the boundary only consists of straight line segments, and the number of the segments is known or only has a limited number of choice.

Under our assumptions, the number of line segments (say N) is known beforehand (we will discuss later on how to decide the number of line segments), remaining problem is therefore how to determine the positions of nodes which connect the line segments which should be as closed as possible to the original boundary. The general idea of our algorithm is to iteratively fix (N-1) number of nodes, find the optimal position for the remaining node. When the number of points in the original boundary data is large, such iterative procedure can take quite a lot of time. In order to speed up, we first perform the gaussian laplacian filter to detect high curvature points, and use these points as the candidate points for nodes. The algorithm flow is illustrated in Fig.4 (located after the "references"). In the following, we give the explanation on the items in Fig.4.

- Gaussian Laplacian Filter

The Gaussian function is given by

$$g(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}} \qquad (6)$$

and the second derivative is equal to

$$g''(t) = C(\sigma^2 - t^2) e^{-\frac{t^2}{2\sigma^2}} \qquad (7)$$

where C is a constant.

Mathematically a convolution may be expressed as:

$$F(x) = f(x) * g(x) \qquad (8)$$

When applying gaussian Laplacian filter g"(t) to the region boundary, we have to calculate the boundary curvature from boundary chain code by the following equation:

$$s(i) = (e(i) - e(i-1) + 11) \bmod 8 - 3 \qquad (9)$$

where e(i) is the Freeman chain codes and s(i) is the curvature elements. s(i) only takes account neighbouring two pixels. After convoluting with g''(t), more pixels in the neighbourhood have been taken into the consideration, which yields more robust result.

- Selecting high curvature points

  This is a thresholding procedure which compare each value from above convolution against certain threshold and high curvature points are selected.

- Determining the position for the first node

  It is obvious that we can simply choose the starting point as the initial position of first node. Because our iteration procedure is much like the optimization by a steepest (gradient) descent, it can very often run into local minimum. To avoid such drawback, we change the initial status of optimization (in our situation, change the position of first node) several times, and compare the results from different initial statuses and choose the best result. Our experiment has proved that it is very efficient method to avoid local optimization for our application.

- Initial partition of boundary

  After the position of first node is decided, the positions of the rest of nodes can be determined along the boundary on the equal interval.

- Optimize the position one by one node

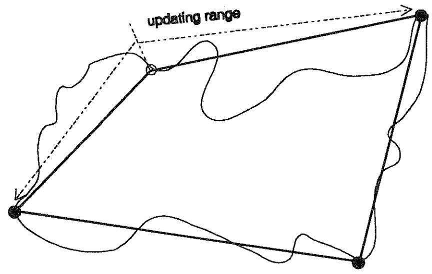

Fig.5

Because we don't use parallel processing, we have to update the position of nodes one by one, as indicated in Fig.5 (filled circle denotes fixed nodes and empty circle represents updating node).

Model selection

So far we have assumed that the number of line segments is known a priori. Such assumptions may be too strict for the method in Fig.4 to be applied into wide range of applications. Let's first relax the assumption of fixed number of line segments to allow this number of segments within a limited number of choice. In this case, we still have no difficulty in applying the method described in Fig.4, only with more rounds, namely, we test each number by this iteration procedure and choose the number under which the difference between the original boundary and modeled line segments reaches the minimum.

For the other type of generic models, like circle, ellipse, the method presented in the fig.4 is not valid any more. A lot of work have been done one this aspect, and the problem is how to integrate the available techniques into segmentation. Nevertheless, our current implementation of the shape modelling is still very useful in a lot of applications such as detecting human-made objects, industrial robots, indoor scene analysis, etc.

6. EXPERIMENT

We test a number of images, based on the principle and algorithms we have described in the previous sections. First image in our experiment is a simulated ideal image added with a gaussian noise (Fig.6a). The minimal difference between the regions is 20, and the standard deviation of the Gaussian noise is 20. We first use Kuwahara filter with 5 pixels of window size to suppress the noise (Fig.6b). In the main algorithm, a 4 thresholding value is used for initial segmentation, which forms the basic small regions, followed by the split-and-merge using only the statistic test (in this case only the mean) with threshold 12. MDL-based operations further groups the regions using the intensity information and shape constraint. After small regions with less than 20 area are removed, the final result is reached, as shown in Fig.6c. Fig.6d shows the region boundaries.

We use the same procedure for the test of the second image (Fig.7), which is the part of a building wall image. a,b,c,d represent the original image, filtered image, segmented image and vectorized image respectively.

7. CONCLUDING REMARKS

In this paper, we have shown how the MDL principle can be adopted in such manner to integrate different knowledge for the purpose of image segmentation. The formulae for encoding image intensity and shape constraints of generic model have been described. In the implementation, we integrate the region growing, curve fitting and model selection in a unified procedure. We feel that it is a good trend which may lead to the general theoretical basis for the segmentation. The extension of this work includes the treatment of complex topological description of object model and integrate with other middle-level image analysis tasks, as indicated in one of our other papers [Zhang].

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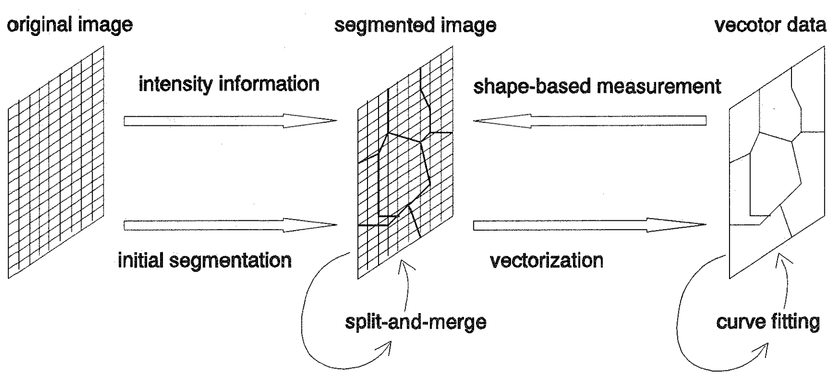

Fig.1

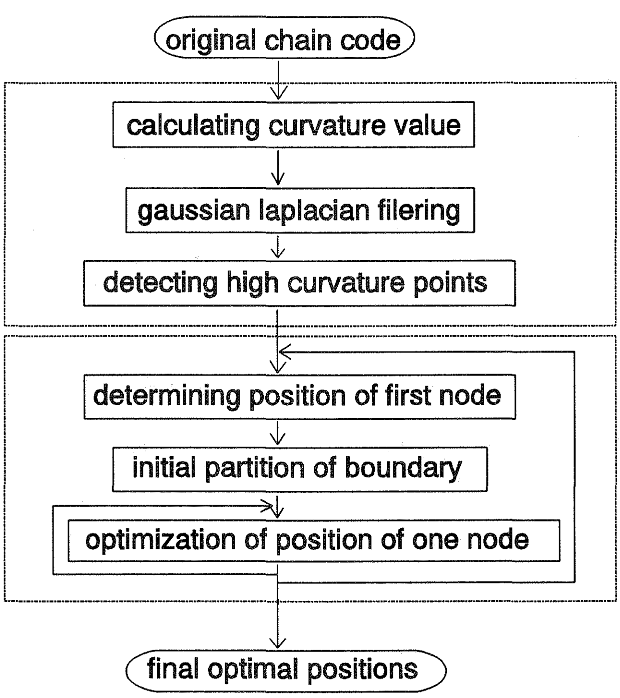

Fig.4

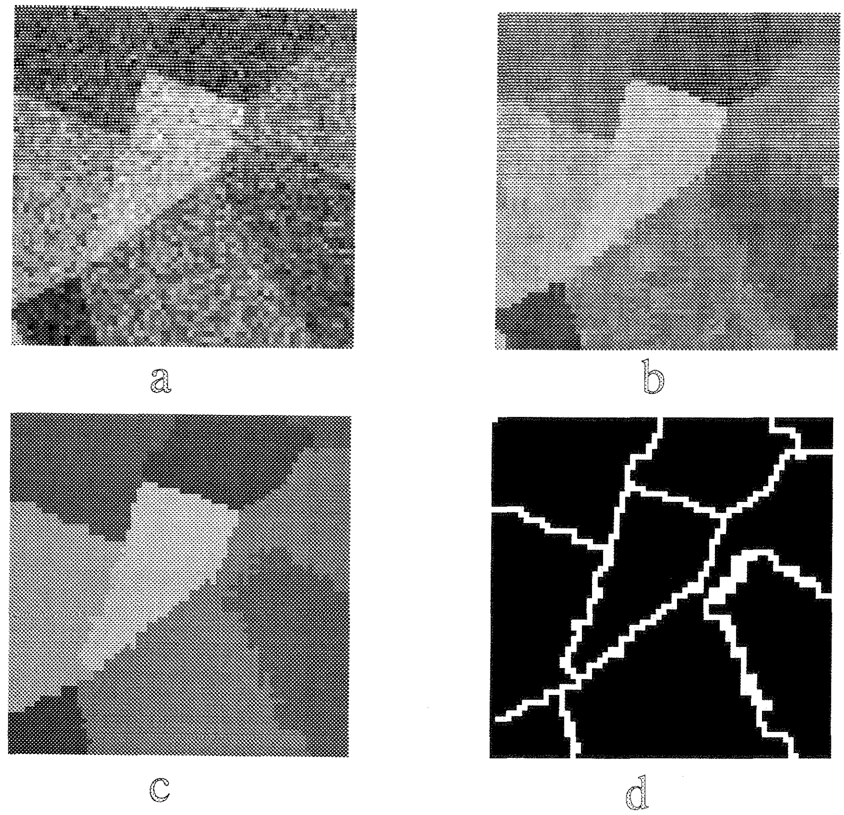

Fig.6

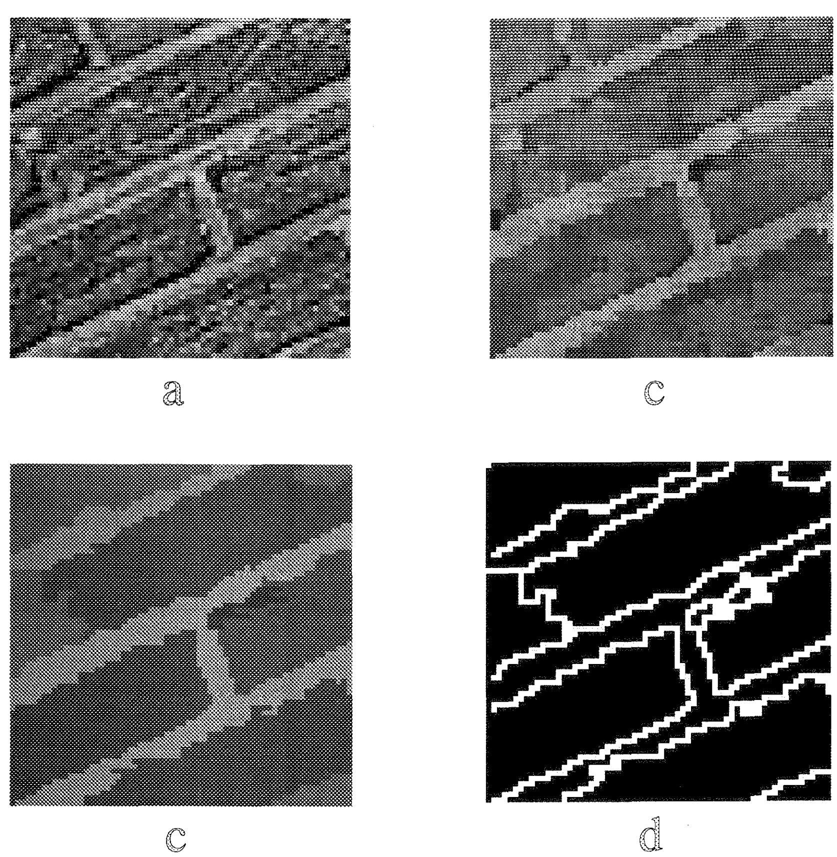

Fig.7