IMAGE SEGMENTATION BASED ON PARAMETER ESTIMATION

Younian Wang
Institute of Photogrammetry and Engineering Surveying
University of Hanover, FR Germany
Commission V

Abstract

In this paper a new method for image segmentation is discussed. It is based on the theories of parameter estimation and hypothesis test of mathematical statistics. The mathematical formulas and criteria of the method are introduced and derived. The function model and random model of the image are discussed. A quantitative index about the separability of the regions are also provided. Some examples are presented to show the effects of the proposed method.

Key Words: image segmentation, parameter estimation, hypothesis test, machine vision, feature extraction.

1. INTRODUCTION

The problems of image segmentation is key important in machine vision. It is also essential for many applications of robot vision in close range photogrammetry. Since the pioneer work of Roberts, Brice and Fennenma[1] much has been written about the topic and many methods have been described in the Literature[2],[1]. The subject continues to receive a great amount of interest because of its essential role in machine vision. It is the basis of many tasks in vision area, e.g. image modelling, image interpretation, scene analysis, image understanding etc.

Mainly speaking, image segmentation indicates to divide an image into several regions according to some consistent principles. It can also be regarded as the classification of the image pixels. The mathematical definition of the image segmentation can be addressed as the following:

For an image R, if m sets of pixels $R_1, R_2, ..., R_m$ exist and:

- $R_i \neq 0$;
- $\bigcup_{i=1}^{m} R_i = R$;
- $R_i \bigcap R_j = 0$;
- each R_i satisfies some consistent principles;
- any combination of the connected sets does not satisfy the above consistent principles;

then the $(R_1, R_2, ..., R_m)$ is called a segmentation of the image R.

There are mainly two types of the consistent principles. The first one supposes there is a homogeneity inside a region, e.g. the same grayvalues. The second one supposes there is a discontinuity between the regions, e.g. the sudden change of the grayvalues. The proposed approaches of image segmetation can be categorized into three kinds, namely threshold-based methods, region-based methods and feature-based methods. The threshold-based methods use one or several grayvalues as the thresholds to divide the image. But in the most cases it is difficult to find the suitable thresholds. The feature-based methods include the edge based methods, which take the edges as the boundaries of the regions, and the classification based methods, which use feature based classifier to determine to which region does a pixel belong. The region based methods involve region growing methods, region splitting and region merging methods, and function approximation methods. Each of the abovementioned methods has its advantages and disadvantages. None of them is perfect and suitable for every case.

In this paper a new method is discussed. It is based on parameter estimation and hypothesis test of mathematical statistics. So that it is theoretically perfect and mathematically well represented. With this method the image noise can be optimally treated. Furthermore the method can also quantitatively describe the separability of the regions. The experiments show that it has also great prospects in applications. Because the images in close range photogrammetry for the applications of machine vision have usually distinct objects and backgrounds, the method should be very suitable in these occassions to distinguish objects from the backgrounds. In the following the methodology is discussed and the mathematical formulas are derived. Its procedures in the applications are analysed and some examples are presented.

2. THEORY OF PARAMETER ESTIMATION

The tasks of parameter estimation are to determine the unknown parameters from the observations. There are several estimation methods for the different models and cases, e.g. point estimation, unbiased estimation, least square method, maximum likelihood method. We introduce here only the least square method for the linear model. If the observations have a normal distribution, the results of the least square method are equal to that of the best linear unbiased estimation and the maximum likelhood method[3].

2.1 Least Square Estimation

If the observations are the linear functions of the unknown parameters, we call it linear model. For the case of the unlinear model we can use the series expasion according to Taylor to obtain the linearized model. So we can discuss the least square method under the linear model without loss of generalization. Suppose we have the function model of parameter estimation as the following:

$$A_{n\times t}\, X_{t\times 1} \; = \; E(L)_{n\times 1} \tag{1}$$

where L is the observations, $E(L)$ is the expectation of L, X is the unknown parameters, n should be greater than t, A is the coefficients matrix and has full rank, and

$$L = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}, A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1t} \\ a_{21} & a_{22} & \cdots & a_{2t} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nt} \end{bmatrix} \tag{2}$$

In the equation (1) A and L are already known, but X and $E(L)$ are unknown.

The differences between observations L and expectations $E(L)$ are called true errors ε, the negative ε is called corrections V:

$$\varepsilon \; = \; L \; - \; E(L), \qquad V = -\varepsilon \tag{3}$$

From the equation (1) we can have

$$V \; = \; AX \; - \; L \tag{4}$$

Suppose the observations L have a normal distribution, the variance of L is Σ, the correspondent weight is P. Then the probability density function of L is

$$f(l_1, l_2, \cdots, l_n) = \frac{1}{(2\pi)^{\frac{n}{2}} \mid \Sigma \mid^{\frac{1}{2}}} exp\{ -\frac{1}{2}[L - E(L)]^T \Sigma^{-1} [L - E(L)]\} \tag{5}$$

and

$$\Sigma = \sigma^2 P^{-1} \tag{6}$$

where σ is called unit weight variance.

The least square method estimates the unknown parameters X under the conditions:

$$V^T PV = minimum \tag{7}$$

The estimates of X then are

$$\hat{X} = (A^T PA)^{-1} A^T PL \tag{8}$$

The estimates of σ and the variance of $\hat{X}$ are

$$\hat{\sigma} = \sqrt{\frac{V^T PV}{n-t}} \tag{9}$$

$$\Sigma_{XX} = \sigma^2 (A^T PA)^{-1} \tag{10}$$

The estimates $\hat{X}$ are the best linear unbiased estimates and the Σ_{XX} is minimal if the observations have a normal distribution.

2.2 Hypothesis Test

By means of hypothesis test we can determine if there are any model errors in the function model (1). Suppose the primary hypothesis is

$$H_0: \quad E(L/H_0) = AX \tag{11}$$

the alternative hypothesis is

$$H_a: \quad E(L/H_a) = AX + HS \tag{12}$$

In the equation (11) and (12) $A_{n\times t}$ and $H_{n\times k}$ are the known coefficients matrixes, $X_{t\times 1}$ are the unknown parameters, $S_{k\times 1}$ are the unknown model error parammeters, $L_{n\times 1}$ are the observations, and

$$L \rightsquigarrow N(E(L), \Sigma) \tag{13}$$

We can rewrite the equation (12) as

$$L + V = AX + HS \quad \text{with weight} \quad P = \sigma^2 \Sigma^{-1} \tag{14}$$

Suppose $n > t + k$, and $[A\,H]$ has full rank, then we can obtain the estimates $\hat{S}$ from the equation (14) with the least square method[4]:

$$\hat{S} = P_{SS}^{-1} H^T PQ_{VV} PL \tag{15}$$

where

$$Q_{VV} = P^{-1} - A(A^T PA)^{-1} A^T \tag{16}$$

$$P_{SS} = H^T PQ_{VV} PH \tag{17}$$

If σ^2 is known, then the following statistic variable T_1 has the uncentralized
χ^2-distribution with k degrees of freedom and uncentralized parameter δ:

$$T_1 = \frac{1}{\sigma^2}(L^T BL) \rightsquigarrow \chi^2(k, \delta^2) \tag{18}$$

where

$$B = PQ_{VV} PHP_{SS}^{-1} H^T PQ_{VV} P \tag{19}$$

$$\delta^2 = \frac{1}{\sigma^2}(S^T P_{SS} S) \tag{20}$$

If the unit weight variance σ is unknown, we can use the following statistic variable T_2:

$$T_2 = \frac{L^T BL}{V^T PV} \rightsquigarrow F(k, n-t-k, \delta^2) \tag{21}$$

If the primary hypothesis is right, the uncentralized parameter δ is zero, so we can use the statistic variable T_1 or T_2 under a certain *risk level* α to determine if there are any model errors.

3. IMAGE SEGMENTATION

3.1 Image Error Analysis

An digital image R is usually described by a two dimentional array G. Each element $g(i,j)$ of the array G represents the grayvalue of the image at the position i−line and j−column. An image pixel can be therefore defined by its position and its grayvalue:

$$p_{ij} = (i, j, g(i,j)) \tag{22}$$

The grayvalue $g(i,j)$ is composed of two parts, namely the true grayvalue $\tilde{g}(i,j)$ and the true error $\varepsilon(i,j)$, which both are unknown:

$$g(i,j) = \tilde{g}(i,j) + \varepsilon(i,j) \tag{23}$$

The error $\varepsilon(i,j)$ of the pixel $p(i,j)$ is caused by the instrument and the enviroments of the image acquisition. There are a variety of influence aspects to the error $\varepsilon(i,j)$, e.g. if a CCD camera is used to acquire a digital image, then the error sources can be among others the lens distortion, the syncronization accuracy, the signal transfer, A/D conversion, the quality of the sensors, the temperature, the humidity and so on[5]. Since each part of the error is quite small and independent, we can suppose it to have a normal distribution according to the central limit theorem of mathematical statistics, that is

$$\varepsilon(i,j) \rightsquigarrow N(\mu,\sigma^2) \tag{24}$$

Usually μ should be zero. If μ is not equal to zero, we can take a simple translational transformation to make μ equal to zero. So the error $\varepsilon(i,j)$ can be supposed :

$$\varepsilon(i,j) \rightsquigarrow N(0,\sigma^2) \qquad (i,j) \in R \tag{25}$$

The equation (25) is also called the random model of the image.

3.2 The Function Model

An image region R_k is defined here as a set of the connected pixels p_{ij}, which true grayvalues $\tilde{g}(i,j)$ satisfy a certain function:

$$f_k((i,j,\tilde{g}(i,j)) = 0 \tag{26}$$

If we really know the light intensity function of an object, we should use this function as the segmentation function. We must linearize the function at first if it has a unlinearized form. Otherwise let us use the first consistent principle for the segmentation, namely suppose there is a homogeneity inside a region. That means the grayvalues in the same region are homogeneous. So we can suppose the grayvalues in a region satisfy a planar equation:

$$\tilde{g}(i,j) = a_k i + b_k j + c_k \tag{27}$$

or more simply a horizontal plane:

$$\tilde{g}(i,j) = x_k \tag{28}$$

Refer to the equation (4), we can set the linear estimation equation as:

$$v(i,j) = a_k i + b_k j + c_k - g(i,j) \qquad (i,j) \in R_k \tag{29}$$

for the whole region we have

$$V = AX - L \qquad \text{with weight } P = I \text{ (unit matrix)} \tag{30}$$

where $X = (a_k, b_k, c_k)^T$.
According the principle of the least square method, we can obtain the optimal estimates of X

$$\hat{X} = (A^TA)^{-1}A^TL \tag{31}$$

and the variance

$$\hat{\sigma} = \sqrt{\frac{L^TL - \hat{X}^TA^TL}{n-3}} \tag{32}$$

3.3 Criterion of Segmentation

We know that n pixels $(p_1, p_2, \cdots, p_n)$ belong to the same region. From the equation (31) and (32) we can get the region parameters $\hat{X}$ and the variance $\hat{\sigma}$. Now the question is how to determine whether the pixel p_{n+1} belongs to the region. If the pixel p_{n+1} belongs to the region, it satisfies the equation (29). Otherwise there is a model error, or we can say the grayvalue g_{n+1} has a different expectation than the grayvalues $(g_1, g_2, \cdots, g_n)$. We can represent it as

$$g_{n+1} + v_{n+1} = a_k i_{n+1} + b_k j_{n+1} + c_k + s_k \tag{33}$$

So we can determine the pixel p_{n+1} by a hypothesis test:

$$H_0: \qquad E(s_k/H_0) = 0 \tag{34}$$

$$H_a: \qquad E(s_k/H_a) = \tilde{s}_k \tag{35}$$

In the equation (18), let $H = (0,0,\cdots,0,1)^T$, $P = I$, we have the statistic variable

$$T_1 = \frac{v_{n+1}^2}{\sigma^2 q_{v_{n+1}v_{n+1}}} \rightsquigarrow \chi^2(1,\delta^2) \tag{36}$$

where v_{n+1} is the correction to g_{n+1} under the primary hypothesis and can be obtained from

$$V = A\hat{X} - L \tag{37}$$

with $V = (v_1, v_2, \cdots, v_n, v_{n+1})^T$, $L = (g_1, g_2, \cdots, g_n, g_{n+1})^T$. $q_{v_{n+1}v_{n+1}}$ is the element of the matrix Q_{VV} in the $n+1$ line and $n+1$ column, which is computed from

$$Q_{VV} = I - A(A^TA)^{-1}A^T \tag{38}$$

where A is same as in the equation (37).

The equation (36) can be simplified as the standardized normal distribution under the primary hypothesis:

$$\omega_1 = T_1^{\frac{1}{2}} = \frac{|v_{n+1}|}{\sigma\sqrt{q_{v_{n+1}v_{n+1}}}} \rightsquigarrow N(0,1) \tag{39}$$

In applications we can use $\hat{\sigma}$ computed from the first n pixels as the σ in the equation (39), we can also use the statistic variable T_2 from the equation (21):

$$t_{n-t-1} = T_2^{\frac{1}{2}} = \frac{|v_{n+1}|}{\hat{\sigma}\sqrt{q_{v_{n+1}v_{n+1}}}} \rightsquigarrow t(n-t-1,0) \tag{40}$$

where

$$\hat{\sigma} = \sqrt{(V^TV - \frac{v_{n+1}^2}{q_{v_{n+1}v_{n+1}}})/(n-t-1)} \tag{41}$$

The others are the same as in the equation (39).

3.4 Separability of Regions

In order to determine which hypothesis is correct, a *risk level* α must be given. This *risk level* α is the probability of incorrect rejection of the primary hypothesis. For a given α, we can find a correspondent *critical value* K_α from the table of the probability density function. If the value of statistic variable (39) or (40) is greater than this K_α, the primary hypothesis is rejected and the alternative hypothesis is accepted. The probability of correct acceptance of the alternative hypothesis is called the *power of test* β, There is another risk in hypothesis test, namely the probability of incorrect acceptance of the primary hypothesis. It is equal to $1-\beta$. The *power of test* β is not only dependent on α (the smaller α the smaller β), but also on the magnitude of the model error s_k.

Now we have to answer the question, how great a model error should be, in order that it can be found by the hypothesis test under the given *risk error* α and *power of test*

β? It is the problem of *separability of regions.* If the pixel p_{n+1} does not belong to the region R_k but to the region R_{k+1}, only if the alternative hypothesis (35) is accepted, the region R_{k+1} can be separated from the region R_k. For a given α and β, we can obtain a minimal uncenterized parameter δ_0,

$$\delta_0 = \delta(\alpha, \beta) \tag{42}$$

For the standardized normal distribution δ_0 is

$$\delta_0 = K_{1-\frac{\alpha}{2}} + K_\beta \tag{43}$$

For example, if $\alpha = 0.1\%$, $\beta = 80\%$, then $\delta_0 = 4.13$. If the value of the statistic variable ω_i is greater or equal to the δ_0, then the model error can be found with the given α and β. So the minimal model error should be

$$s_i^0 = |v_i^0| = w_i^0 = \sigma \frac{\delta_0}{\sqrt{q_{v_i v_i}}} \tag{44}$$

That means, if and only if the difference of the grayvalues between the region R_k and R_{k+1} should be equal or greater than s_i^0, the two regions can be separated from each other with the given *risk error* α and *power of test* β.

If the number of the observations n is great enough, the $\sqrt{q_{v_i v_i}} \approx 1$, so in this case the equation (44) can be approximated as:

$$s_i^0 \approx \sigma \delta_0 \tag{45}$$

4. IMPLEMETATION OF SEGMENTATION

4.1 Choice of Start Area

For the different function models of a region we must choose a start area with certain number of pixels, e.g. for the planar function (27) the start area should have three pixels. We use the procedure of minimal difference in grayvalues (MDG) to set up the start area:

- to find an unclassified pixel as the first one;
- to choose the pixel with MDG to the first pixel among all the neighbouring pixels;
- the next pixel should have MDG to the average grayvalue of the choosen pixels.

The neighbouring pixel can stay in the 4 directions or 8 directions. We should pay attention to the case when all the pixels in the start area are in the same line. In this case the solution of the equation (31) are indefinite. As an alternative we can then use the function model (28).

How to choose the first pixel is also quite important. It may have better results if the first pixel does not stay on the boundary of two regions.

4.2 Labelling of Regions

In order to keep the implementation as quick as possible, the labelling of the pixels should be optimized. The undetected pixels, the rejected pixels and the accepted pixels can be separatly labelled in order to avoid repeatedly searching. By searching for the acceptable pixels the rules about priority of the nearest neighbouring pixel (lengthwise priority) and the shortest distance to the start pixel (crosswise priority) can be used. After a region has been segmented, all the pixels in the region will be also identified and specially labelled.

After all the regions have been segmented, the boundaries of them can be also easily obtained.

4.3 Handling the Small Areas

If two regions have a great difference in grayvalues, then the grayvalues between two sides of the boundary do not change suddenly, but gradually. So sometimes there may be a small region along the boundary found. These kinds of small regions are usually very narrow and superfluous. According to their properties we can merge them. Here we can also use the previous knowledge if there is any.

4.4 Some Examples

In order to examine the effectiveness of the method we have carried out several experiments. The results of them are illustrated in the following figures.

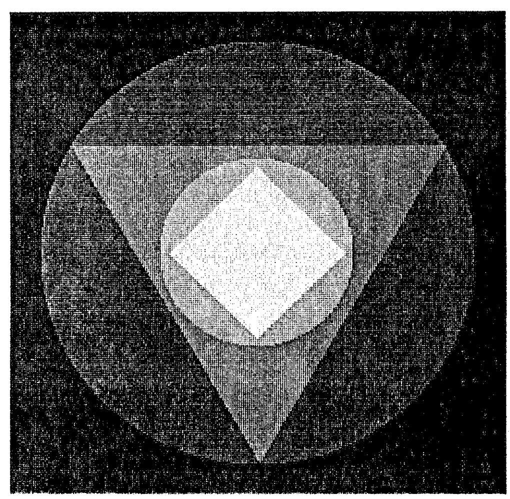

Figure 1: the first image with $\sigma = 5$

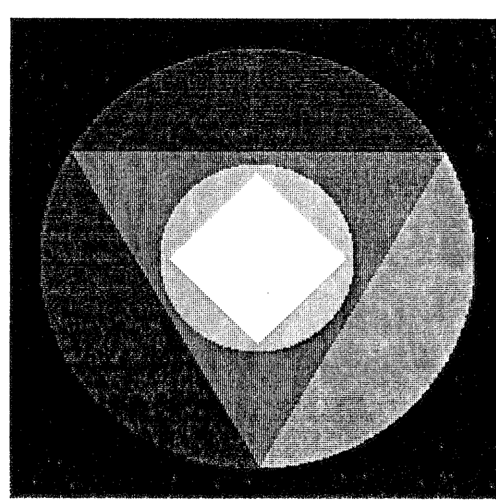

Figure 2: the segmented results of Fig. 1

Figure 1 is a simulated image with 5 regions. The range of the grayvalues of the image is from 0 to 255. The difference of the average grayvalues of two neighbouring regions is about 50. A Gaussian noise with the expectation $E(\varepsilon) = 0$

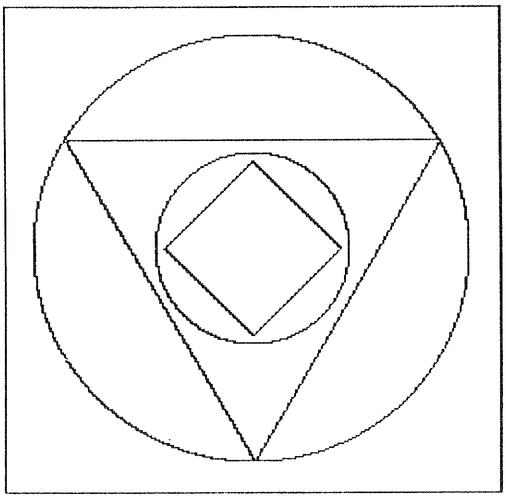

Figure 3: the regions boundaries of Fig. 1

and the variance $\sigma = 5$ is added to each pixel of the image, and the maximal difference of the noise is 40. With the *risk error* $\alpha = 0.1\%$ both function models (27) and (28) are tested. The segmented results of the planar function model (27) are shown in the Figure 2. From the figure 2 we can see that the lower-right part of the large circle is segmented as an independent region, because this part is isolated from the two other parts by the triangle. The segmentation here is entirely correct. The boundaries of the segmented regions are shown in the figure 3. The results of the horizontal plane function model is same, except that there is a very small unnecessary region on the boundary between the triangle and the small circle, which is caused by the large noise and can be easily eliminated.

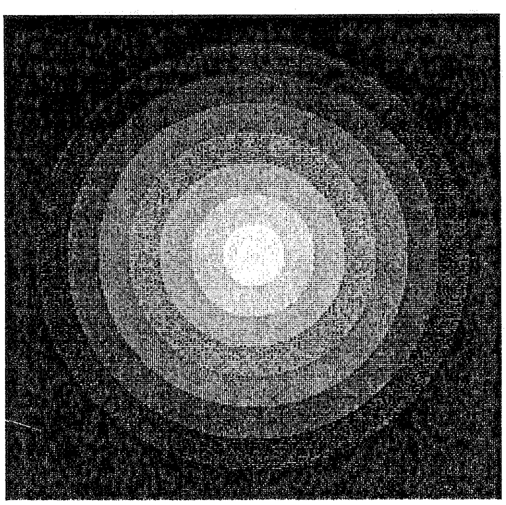

Figure 4: the second image with $\sigma = 8$

Figure 4 is also a simulated image with normally distributed errors. It has 8 regions and the difference of the avarage grayvalues of two neighbouring regions is about 32. The variance of the Gaussian noise is $\sigma = 8$, and the maximal difference of the noise is about $65(\pm 4\sigma)$. That means the grayvalues between two neighbouring regions have already overlapped. We have also tested two function models for the image. By *risk error* $\alpha = 0.1\%$ both models can only segment the image into three regions, because the regions can not separated in this case. By $\alpha = 1\%$ both the models can divide the image correctly except that there is a small

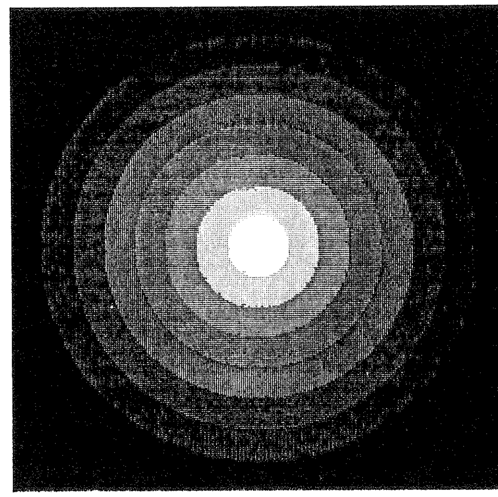

Figure 5: the segmented results of Fig. 4

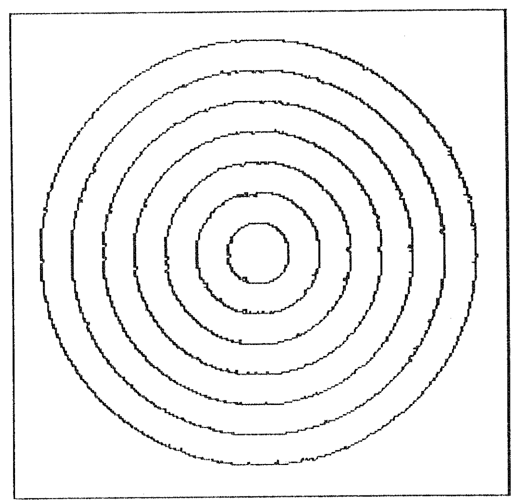

Figure 6: the regions boundaries of Fig. 4

unnecessary region as in the first image. From the equations (43) and (44) we know that by larger α the separability of the regions increases too. The segmented results and the boundaries are shown respectively in the figure 5 and 6.

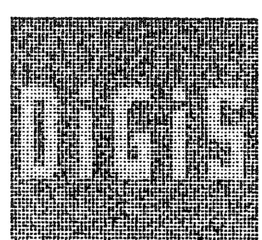

Fig. 7 the 3. image with $\sigma = 20$

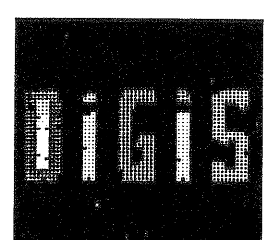

Fig. 8 the coarse segmentation

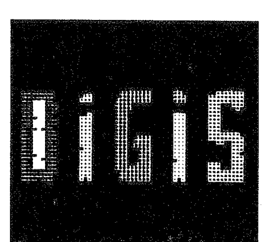

Fig. 9 results after region-merging

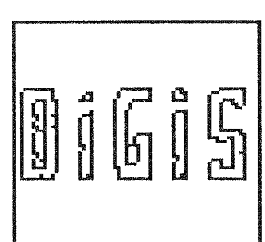

Fig. 10 regions boundaries of Fig. 7

Figure 7 is a test image from the OEEPE-test on the feature-based segmentation. The image is 64×64 pixels large and has the Gaussian noise with the variance $\sigma = 20$. The segmented results for both models are nearly the same, but by $\alpha = 0.1\%$ the part inside the letter D is treated as three small regions for the horizontal plane function model and the planar function model segments this part into the same region as the letter D itself. Figure 8 shows the segmented results of the horizontal function model, figure 9 is the results after the region-merging from figure 8, and figure 10 the boundaries graph.

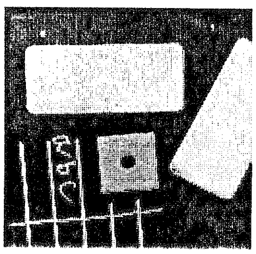

Fig.11 the 4. image from CCD camera

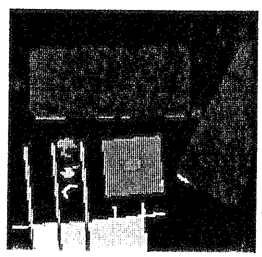

Fig. 12 the coarse segmentation

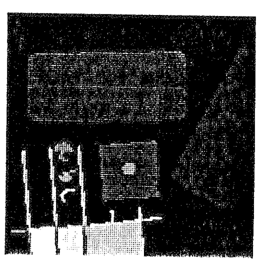

Fig. 13 results after region-merging

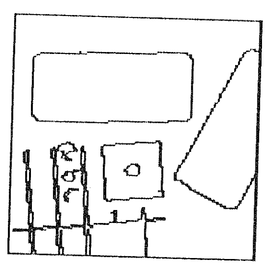

Fig. 14 regions boundaries of Fig. 11

Figure 11 is a real image taken by a CCD camera. The image has 128×128 pixels and the range of the grayvalues is from 0 to 255. We can see from the segmented results figure 12 that there are a few small regions at the boundaries. After the elimination of the small regions we have the results shown in the figure 13. Figure 14 shows again the boundaries of the regions.

5. CONCLUSIONS

From the above discussions and examples we can see the method of image segmentation based on parameter estimation has not only a perfect theoretical and mathematical basis, but also great prospects in applications. With this method we can treat the image noise quite perfectly. We can also quantitatively judge the separability of the regions. The results of the simple horizontal plane function model are not worse than the planar function model, but the former works much faster. Usually there will be more regions segmented with the larger *risk error* α.

A further work is how better to solve the problem of the prevention and mergence of the superfluous small regions at the boundaries. It may be better solved together with the edge-based method or by using an iterative approach. More experiments should be carried out on the real images in order to examine the effects of the method and to evaluate which function model and which *risk error* α are more suitable.

REFERENCES

1. Sanz, J.L.C., 1989, Advances in Machine Vision, Springer Verlag.

2. Boyle, R.D., 1988, Thomas, R.C., Computer Vision, Blackwell Scientific Publications LTD.

3. Koch, K.R., 1980, Parameterschätzung und Hypothesentests in linearen Modellen, Dümmler Verlag, Bonn.

4. Li, D., 1985, Theorie und Untersuchung der Trennbarkeit von groben Paßpunktfehlern und systematischen Bildfehlern bei der photogrammetrischen Punktbestimmung, Dissertation, University of Stuttgart.

5. Grün, A., 1988, Towards Real-Time Photogrammetry, Photogrammetia(PRS), 42.

6. Maitre, H., et al., 1990, Range Image Segmentation based on function approximation, Close Range Photogrammetry Meets Machine Vision, SPIE Volume 1395.

7. Wang, Y., Jacobsen, K., 1991, Model Based Fast Recognition of the Structure of Industrial Workpieces, In B. Radig (editor), Mustererkennung 1991, Informatik Fachberichte 290, Springer Verlag.