ON-LINE TRIANGULATION FOR AUTONOMOUS VEHICLE NAVIGATION

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PURPOSE:

At the Center for Mapping of The Ohio State University, a mobile mapping workstation has been built which integrates a GPS receiver, an inertial system, and a stereo-vision system for the automatic capture of highway data. Digital stereo-images are captured together with vehicle positions at regular increments. They can be connected into a strip by applying bundle triangulation. This paper focuses on the development of an algorithm to utilize the stereo-vision system in sequential triangulation mode. This approach can be directly applied to intelligent vehicle highway systems(IVHS) to automatically navigate a vehicle along a road.

Recursive problems, such as on-line triangulation require algorithms providing real-time responses. Givens Transformations offer an ideal solution to such problems as they yield a direct solution to linear least-squares problems without forming the normal equations and permit continuous monitoring of the solution vector and necessary statistical measures. Givens Transformations without square roots require significantly less operations than conventional Givens. In this paper the mathematical formulation is given, followed by a description of the triangulation program developed for this study. Various implementation problems are addressed and results of a practical example are presented.

KEY WORDS: Givens, GPS, Triangulation, IVHS, Navigation.

INTRODUCTION

Various sensors have been utilized independently for the collection of data for Geographic Information Systems(GIS) and for the production of digital maps. The Global Positioning System(GPS) can produce three dimensional ground coordinates by triangulation using measurements to a minimum of four satellites. Overlapping stereo-images are commonly used to map surface details and for the extraction of elevations. Inertial systems, when carried in a vehicle, continuously record velocity and acceleration changes and thus can be applied to the surveying of linear features such as roads. During the past two years, research conducted by the **Center for Mapping** at **The Ohio State University**, focused on the integration of a GPS receiver, an inertial system, and a stereo-vision system into a van for the automatic recording of data from the highway environment[Novak, 1990;Bossler,et.al., 1991]. The prototype of this vehicle appears in Plate 1.

An important problem associated with GPS occurs when the satellite signals are unable to reach the receiver antenna. This problem, called "loss-of-lock", occurs when obstructions such as buildings, trees, and bridges come between the satellite and the antenna. The inertial system can be used to overcome this problem. It can provide position information during loss-of-lock periods, knowing the GPS coordinates of the vehicle before and after the satellite signal was interrupted.

The stereo-vision system offers another solution to the same problem. Stereo-pairs captured during loss-of-lock can be tied together and controlled by the GPS coordinates of the van before and after signal loss. Utilizing on-line triangulation techniques, a strip of stereo-pairs can be formed in a sequential manner. This is advantageous because of the sequential nature of the data collection. Thus, the vehicle can literally be "navigated" from one stereo-pair to the next. In this article, we investigate the feasibility of using visual navigation to bridge over areas without satellite lock.

Before a photogrammetric solution of this navigation problem can be derived, it is necessary to differentiate between sequential and simultaneous adjustment solutions. The appropriate simultaneous solution for strip triangulation is the well known bundle adjustment. This method requires all data collection to be completed before the adjustment. The sequential solution allows the strip to be built model by model, and, additionally, permits the editing of data at any stage of the process.

The very definition of "on-line triangulation" requires that results be available at any time during processing. Due to the sequential nature of the measurement process, the use of sequential algorithms follows naturally. As on-line triangulation is an interactive process between the operator and the computer, real-time or near real-time responses are necessary. Thus, there has been a continuing search for fast and efficient algorithms for sequential adjustment.

The most prominent approaches are the "Kalman-form", which updates the inverse of the normal equations [Mikhail,Helmering,1973], the "Triangular Factor Update" (TFU), that updates the factorized normals directly [Gruen,1982], and Givens Transformations which updates the factorized normal equation matrix [Blais,1983]. Our algorithm is based upon Givens Transformations.

The Givens method is an orthogonal transformation technique based on the use of plane rotations to annihilate matrix elements. This approach provides a direct way for solving linear least-squares problems without forming the normal equations. Since all updating is done in the factorized normals, numerical instabilities associated with forming and solving the normal matrix are avoided. Large, sparse design matrices are readily exploited by Givens Transformations.

Pertinent to the on-line triangulation problem, Givens Transformations process one row of the design matrix at a time, and thus can be used for the sequential addition or deletion of observations in an interactive environment. Additionally, the solution vector is available at any stage by simply performing a back substitution in an upper triangular equation system. Givens Transformations can also be used to update the cofactor matrix simultaneously with the solution, thus providing the necessary statistical measures for analysis. This method adapts easily to handle weighted observations [Gentleman, 1973; Blais, 1983] in addition to weighted parameters.

Plane rotations are usually designed to annihilate only one element at a time. The inherent weakness is that for each annihilated element, one square root is required, and additionally, for each pair of elements, four multiplications are needed. For this reason, Givens Tranformations were not considered as a viable solution when results are required in real-time. However, this changed when Gentleman[1973] proposed a modification which eliminates the square roots, reduces the number of multiplications by one-fourth, and facilitates weighted least squares at no extra cost.

In this study, Givens Transformations without square roots are used for the on-line triangulation of strips of sequential stereo-pairs obtained by the mapping van. The program developed to solve this problem allows for a variable size parameter vector, the update of the solution vector, and the ability to perform a simultaneous solution at the operators convenience.

Plate 1: Mapping Van of the Project Application of the Global Positioning System for Transportation Planning

A brief overview of the mapping van project is followed by the mathematical formulation of conventional Givens Transformations and Givens without square roots. We then describe the implementation of Givens Transformations for sequential navigation of stereo-pairs. The results of the triangulation of a simulated strip of stereo-pairs are presented. Finally, we close with conclusions and future research plans.

THE MAPPING VAN PROJECT

The project *Application of the Global Positioning System for Transportation Planning* has been coordinated by the **Center for Mapping** at the **Ohio State University.** This project was initiated in September of 1990 with the goal of developing a mobile van workstation for the automatic mapping and recording of highway alignments and other features which in turn can be entered into a GIS. A working prototype was completed in February of 1991 integrating a GPS receiver, a gyro-based inertial system, two wheel count sensors, and a stereo-vision system which includes two high resolution digital cameras and an image processing system with high-capacity data storage capabilities.

The positioning portion of this project is based on two GPS receivers used in differential mode. One receiver remains stationary at a base while the other is mounted on the van for mobile data collection. All point coordinates from the mobile receiver are determined relative to the base station. The inertial system provides information of position change in terms of direction, pitch, roll, and distance traveled. Because the obstruction of GPS signals is a common problem, the inertial system is an invaluable link during periods of satellite signal loss. The inertial system alone can track the van position to better than one meter for each mile traveled. The GPS van positions, accurate within one to three meters, are used as control and the positional change information from the inertial sensors are fitted to them in a least-squares adjustment.

The stereo-pairs obtained by the vision system can serve many purposes. Each stereo-pair, along with its geographic location can form an important component of a GIS. They can be utilized with image matching and feature extraction techniques for precisely locating features such as center and edge lines of roads and traffic signs. Future research will focus on the survey of bridges and overpasses and the evaluation of road surfaces for cracks and pavement stress [Novak, et.al., 1991]. In this paper however, we will show that the stereo-vision system, similar to the inertial system, can provide a solution to the positioning problem when satellite signals are not available. The images captured during a loss-of-lock period can be tied together by conjugate points. The strip of stereo-pairs can be sequentially triangulated using the GPS van coordinates just before and after loss of signal for control, as seen in Figure 1.

A sequential solution is advantageous here over a simultaneous solution for two reasons: first, the stereo-pairs are captured in a sequential manner; second, the vehicle can be literally "navigated" from one stereo-pair to the next. Thus the current position of the vehicle is always known during loss-of-lock periods. At this time the triangulation has to be performed by an operator. It is anticipated, however, that by utilizing linear features(e.g. road edges) this procedure could be automated and performed in real-time [Novak, 1990].

Because the geometry of the vision system is assumed to be stable during operation, an accurate calibration of the entire system must be performed before any data collection. For this calibration, the interior and relative orientations of the two cameras, and the location of the GPS antenna relative to the cameras are determined using a three dimensional test field of retro-reflective targets. Local coordinates obtained by the vision system can be immediately transformed into a global system. The reference is given by the GPS antenna and the orientation of the van as defined by the inertial measurements.

MATHEMATICAL FORMULATION OF GIVENS TRANSFORMATIONS

In this section, the algorithm we used for sequential strip formation is explained. It is based on Givens Transformations and Givens Transformations without square roots. As estimation model, the Gauss-Markov model is used.

The QR Decomposition

Given an n x 1 coefficient vector l and a m x n design matrix A such that m ≥ n, the problem is to compute the n x 1 parameter vector x so as to minimize the sum of the squares of the elements of the m x 1 residual vector v defined by

$$v = A\,x - l. \tag{1}$$

Considering unweighted observations, the solution is given by

$$\hat{x} = (A^T A)^{-1} A^T l. \tag{2}$$

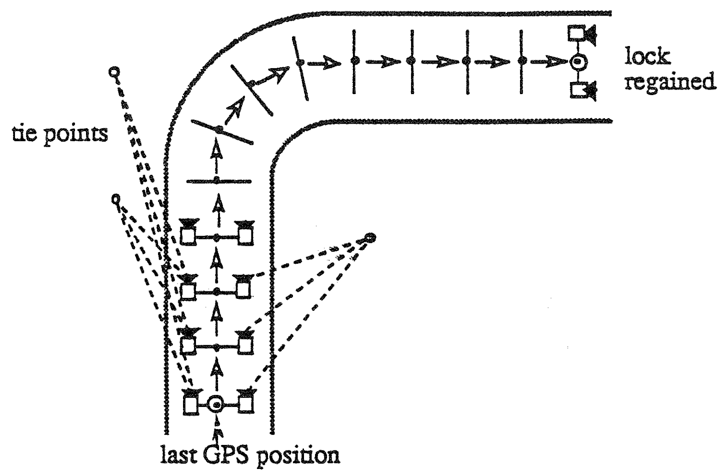

Figure 1: A Strip of Stereo-pairs Formed by Sequential Triangulation to Bridge Over Areas Without Satellite Signals.

This solution is normally obtained by first factorizing the normals(A^TA) into the product of a lower triangular(L) and upper triangular matrix(U) resulting in the system

$$L\, U\, \hat{x} = b. \tag{3}$$

By letting $d = L^{-1}\, b$, the system becomes

$$U\, \hat{x} = d \tag{4}$$

and can be solved by a simple back substitution.

If, however, the decomposition

$$A = Q\, R \tag{5}$$

is available, where Q is a m x n matrix whose columns are orthonormal and R is a n x n upper triangular matrix, the normal equations can be written as

$$R^T\, Q^T\, Q\, R\, \hat{x} = R^T\, Q^T\, l. \tag{6}$$

Since Q is orthogonal, $Q^TQ = I$ and additionally, since R is nonsingular if A^TA is nonsingular, we get

$$R\, \hat{x} = Q^T\, l. \tag{7}$$

One can see that (4) and (7) are equivalent where $U = R$ and $d = Q^Tl$. The solution to this system can be obtained without forming the normal equation matrix(A^TA), thus avoiding the instabilities associated with its formation.

As only **d** is needed for the solution, **Q** is not explicitly required [Gentleman,1973]. Obtaining **R** and **d** is a matter of applying a series of Givens Transformations to **A** and **l**.

If the design matrix **A** is associated with a weight matrix **P**, the solution is given by

$$\hat{x} = (A^T\, P\, A)^{-1}\, A^T\, P\, l. \tag{8}$$

For uncorrelated observations **P** is a diagonal matrix and the design matrix **A** can be premultiplied by $P^{½}$. The QR decomposition is then applied to this modified design matrix. If the observations are correlated, **P** is fully populated and positive definite, and can be factorized by the Cholesky method into the product of a lower and upper triangular matrix,

$$P = L\, L^T = U^T\, U. \tag{9}$$

In this case matrix **A** is premultiplied by **U** before the **QR** decomposition is performed.

<u>Sequential Estimation</u>

In many photogrammetric applications, new measurements must be added to a system once a solution is computed. In such a case it is of advantage to directly update the reduced normal equation matrix R and avoid a full solution of the new system. Therefore, operations are needed to add, delete, or replace observations or to impose constraints. All of these operations can be based on Givens transformations, as explained below. The development below was proposed by [Gruen, 1985].

At stage *k-1* the reduced system takes the form of **(4)**. The addition of one observation equation including a set of new unknown parameters leads to the following form(stage *k)*

$$\begin{bmatrix} R \\ \\ a^T_{(k)} \end{bmatrix} \begin{bmatrix} x \\ --- \\ y \end{bmatrix} = \begin{bmatrix} d \\ --- \\ l_{(k)} \end{bmatrix} \tag{10}$$

where $a^T_{(k)}$ is the new coefficient vector, y is the new parameter vector of length p, and $l_{(k)}$ is the right hand side of the new observation equations. Applying a series of n (number of total system parameters) Givens Transformations

$$Q = Q_n\, Q_{n-1}\, \dots\, Q_1, \tag{11}$$

to **(10)** gives

$$Q \begin{bmatrix} R & 0 \\ 0 & 0 \\ a^T_{(k)} & \end{bmatrix} \begin{matrix} \}n-p \\ \}p \\ \}1 \end{matrix} = \begin{bmatrix} \dot{R} \\ \\ 0 \end{bmatrix} \begin{matrix} \}n \\ \\ \}1 \end{matrix}, \tag{12}$$

$$Q \begin{bmatrix} d \\ 0 \\ l_{(k)} \end{bmatrix} \begin{matrix} \}n-p \\ \}p \\ \}1 \end{matrix} = \begin{bmatrix} \dot{d} \\ \\ \dot{l}_{(k)} \end{bmatrix} \begin{matrix} \}n \\ \\ \}1 \end{matrix}. \tag{13}$$

The updated solution vector is found by backsubstituting into

$$\dot{R}\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \dot{d}. \qquad (14)$$

Practically, from Equation **(10)**, we consider one row vector from the system **Rx = d** and an observation row vector from the system **Ax = l**,

$$\begin{matrix} 0 \dots 0 \; r_i \; r_{i+1} \dots r_k \dots \\ 0 \dots 0 \; a_i \; a_{i+1} \dots a_k \dots \end{matrix} \qquad (15)$$

A single Givens Transformation rotates these two vectors and replaces them with

$$\begin{matrix} 0 \dots 0 \; r'_i \; r'_{i+1} \dots r'_k \dots \\ 0 \dots 0 \; 0 \quad a'_{i+1} \dots a'_k \dots \end{matrix} \qquad (16)$$

where

$$\begin{aligned} r'_k &= c\, r_k + s\, a_k \\ a'_k &= -s\, r_k + c\, a_k \\ c^2 + s^2 &= 1. \end{aligned} \qquad (17)$$

The requirement that $\mathbf{a}_i$ be annihilated to zero allows the computation of the rotation parameters:

$$\begin{aligned} r'_i &= \sqrt{r_i^2 + a_i^2} \\ c &= r_i \,/\, \sqrt{r_i^2 + a_i^2} = r_i \,/\, r'_i \\ s &= a_i \,/\, \sqrt{r_i^2 + a_i^2} = a_i \,/\, r'_i \end{aligned} \qquad (18)$$

from the diagonal elements of **R** and the corresponding elements of the coefficient vector. Zeros appearing in corresponding elements of both vectors are left unchanged.

R and the right hand side **d**, after being augmented by a new observation vector, appear as in Figure 2.

$$\begin{matrix} r_{11} & r_{12} & r_{13} & \dots & r_{1n} & d_1 \\ & r_{22} & r_{23} & \dots & r_{2n} & d_2 \\ & & r_{33} & \dots & r_{3n} & d_3 \\ & & & \dots & \dots & \dots \\ & & & \dots & r_{nn} & d_n \\ & & & & & \Omega \\ a_1 & a_2 & a_3 & \dots & a_n & l \end{matrix}$$

Figure 2: R Matrix Augmented By New Coefficient Vector

An additional element (Ω) is added to the right hand side to maintain the root residual sum of squares

$$\Omega = \sqrt{v^T v} = d^T d. \qquad (19)$$

as was shown by [Lawson and Hanson,1974]. Thus, the variance factor

$$\hat{\sigma}_0^2 = \frac{\Omega^2}{r}, \qquad (20)$$

can be updated with Givens Transformations together with R and d.

Computations Without Square Roots

By using conventional Givens Transformations, one needs approximately ***2n²*** multiplications and ***n+1*** square roots to process each new observation vector [Gentleman, 1973]. This is unacceptable for real-time applications. An alternative implementation of Givens Transformations avoids the square roots and requires only three-fourths as many multiplications.

This method involves finding a diagonal matrix **D** and a unit upper triangular matrix $\bar{\mathbf{R}}$ such that

$$R = D^{\frac{1}{2}}\, \bar{R}. \qquad (21)$$

A row of the product $\mathbf{D}^{1/2}\bar{\mathbf{R}}$ is rotated with a scaled row of **A** [Gentleman,1973],

$$\begin{matrix} 0 \dots 0 \; \sqrt{d} \quad \dots \sqrt{d}\; \bar{r}_k \dots \\ 0 \dots 0 \; \sqrt{\delta}\; a_i \dots \sqrt{\delta}\; a_k \dots \end{matrix} \qquad (22)$$

where **d** is the diagonal element of the matrix **D** and δ is the scale factor for the coefficient vector and is initially set to one.

After one rotation, from Equations **(17)** and **(18)**, the newly transformed rows are

$$\begin{matrix} 0 \dots 0 \; \sqrt{d'} \dots \sqrt{d'}\; \bar{r}'_k \dots \\ 0 \dots 0 \quad 0 \quad \dots \sqrt{\delta'}\; a'_k \dots \end{matrix} , \qquad (23)$$

where **d′** is the updated diagonal element, δ′ is the updated scale factor and

$$\begin{aligned} d' &= d + \delta\, a_i^2 \qquad (24) \\ \delta' &= d\,\delta \,/\, (d + \delta\, a_i^2) = d\,\delta \,/\, d' \\ \bar{c} &= d \,/\, (d + \delta\, a_i^2) = d \,/\, d' \\ \bar{s} &= \delta\, a_i \,/\, (d + \delta\, a_i^2) = \delta\, a_i \,/\, d' \\ a'_k &= a_k - a_i\, \bar{r}_k \\ \bar{r}'_k &= \bar{c}\, \bar{r}_k + \bar{s}\, \bar{a}_k. \end{aligned}$$

Practically, **D** and $\bar{\mathbf{R}}$ are initialized to zero and the scale factor δ is initialized to one. After one transformation, the updated rows are expressed as a row of an updated $\mathbf{D}^{1/2}\bar{\mathbf{R}}$ and an updated scaled row of **A** with a new scale factor.

This method simplifies the weighted least squares problem, for which each observation equation is multiplied by the square root of its weight. Using Givens Transformations without square roots, this is accomplished by simply initializing the scale factor to the weight instead of one.

Introducing the same observation into the a system several times with various positive and negative weights, has the same effect as introducing it only once with the sum of the weights. Therefore, observations can be deleted by simply reintroducing them with the negative of their previous weight [Gentleman, 1973].

IMPLEMENTATION FOR ON-LINE TRIANGULATION OF STEREO-PAIRS

The method of Givens Transformations without square roots was implemented in an on-line triangulation program to form a strip of stereo-pairs captured by the stereo-vision system of the mapping van. This program allows for a variable size solution vector in order to add new unknowns to the system, continuous monitoring of the solutions and the cofactor matrix, the capability of a simultaneous solution at the operators convenience, and the imposition of constraints to fix certain parameters, such as the relative orientation of any stereo-pair.

The Triangulation Program

The triangulation functions and interface programs were implemented on a workstation, and allow for interactive image display and point selection with a cursor on the screen. Sequential strip triangulation begins when the operator displays the first stereo-pair on the computer screen. Three dimensional coordinates of the perspective centers of the left and right cameras are known from the GPS position of the van and the calibrated relative orientation of the camera pair. Conjugate points are measured in both images manually or by using an image matching technique. After measuring a few points in the first stereo-pair we can compute the exterior orientation parameters with the base and the relative orientation constrained.

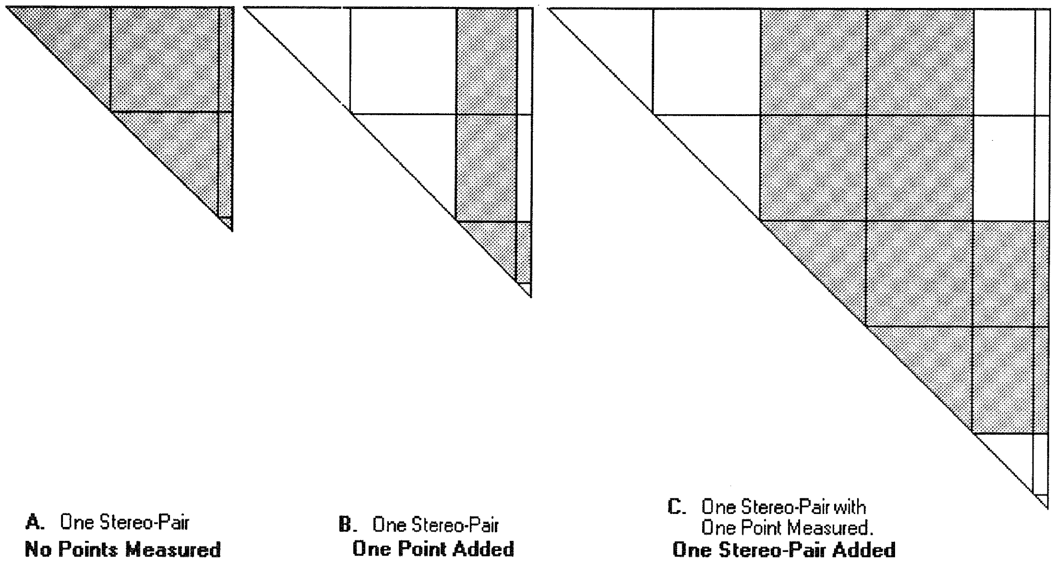

Figure 3: Changes in Matrix Structure as Points and Stereo-Pairs are Added

At this point the datum is established by fixing seven parameters. Typically, these are the coordinates of the perspective centers and one camera rotation angle. When additional stereo-pairs or new points are added to the system, memory for the upper triangular matrix, its inverse, and the solution vector is allocated accordingly, and approximate values for new parameters are computed. To avoid singularities, a minimum of five points must be measured for each additional stereo-pair. The determination of approximate values is addressed later.

When points are measured, the observations and residuals appear on the screen. The solution vector, corresponding precision values and the variance factor are visible to the operator upon request. A simultaneous solution can be performed anytime at the operators convenience.

Variable Size Parameter Vector

Two types of parameters can be added to the system; the twelve orientation parameters of a new stereo-pair or the three coordinate parameters of an additional point. For both types of additions, the matrix storing **D** and $\bar{\mathbf{R}}$ increases in size. To better illustrate how this matrix structure changes when stereo-pairs or points are added, Figure 3 can be used.

The shaded portions in Figure 3 represent additions to the matrix structure. This is also indicated in the bold type in the caption under each matrix. In Figure 3A memory has been allocated for the initial stereo-pair. This is a 13 x 13 matrix where columns 1-12 are devoted to the orientation parameters of the left and right cameras. Column 13 contains the right hand side, and element (13,13) maintains the residual sum of squares.

In Figure 3B we still have one stereo-pair but one new point has been measured. This adds three rows and three columns to the original matrix structure to account for the three dimensional coordinates of the point. The matrix is now 16 x 16.

In Figure 3C, one stereo-pair is added to the matrix structure in Figure 3B. This adds 12 rows and 12 columns for the orientation elements of the second stereo-pair to the matrix structure. The matrix has grown to 28 x 28.

Adding a new point or stereo-pair to the matrix structure is not simply a matter of allocating the proper amount of memory. The triangular matrices used in this program are stored as one dimensional vectors. When additional memory is allocated to an existing matrix, it is simply tacked onto the end of that vector, leaving all previous elements in their original memory locations. As can be seen in Figures 3B and 3C, the correct position for the newly allocated memory actually lies within the vector, not at the end. Therefore, all original elements must be shifted to maintain their correct location.

Approximate Values

The choice of approximate values can be a problem with sequential estimation in a non-linear system. Sequentially updating the parameters is not a viable solution unless the entire system is re-linearized. Otherwise, if the initial values were not good, the parameter vector would "drift off" [Gruen, 1985]. One answer to this problem is simply to start with good enough approximate values. This however, may not always be possible. The more practical solution is to perform a simultaneous solution periodically. We follow this approach in our implementation.

For the first stereo-pair in a strip, the true coordinates of the perspective centers are determined by relating the position of the cameras to the known coordinates of the GPS antenna and utilizing the information from the inertial system for orientation. This information is available for each stereo-pair as long as satellite lock is maintained and is stored in a feature file. For stereo-pairs during the loss-of-lock period, approximate values for are determined from the distance and direction traveled from the previous stereo-pair. This information is available from the inertial system or can be estimated from the speed of the vehicle. Approximate values for the coordinates of new points are obtained by intersecting light rays from the two perspective centers of the camera-pair.

Control

For a strip of stereo-pairs obtained by the mapping van, there are no control points along the road. The datum is defined by the known GPS positions of the van before and after a loss-of-lock period, and from the orientation information at these positions, which is available from the inertial system.

	Xp [mm]	Yp [mm]	C [mm]	ω [rad]	ϕ [rad]	κ [rad]
left	0.050	0.030	6.80	0.00	0.00	0.00
right	0.030	0.050	6.90	0.10	0.10	0.10

Table 1: Simulated Interior and Relative Orientations of the Vision System Cameras

Practically, the perspective center coordinates and one camera rotation angle for the first stereo-pair are fixed by using pseudo-observations with high weights. This is done by setting the corresponding diagonal elements of matrix **D** to a high weight. Points measured in the first stereo-pair then, are determined by intersection. Point coordinates and orientation parameters for all additional stereo-pairs are computed using the full bundle method. When lock is regained the GPS position of the van will not exactly correspond to the position computed by sequential triangulation. Corrections must be applied to all perspective centers and orientation parameters between the first and last stereo-pair. This is accomplished by fixing the orientation parameters of the last stereo-pair just as those of the first stereo-pair are fixed, and then performing a simultaneous adjustment.

Implementation of a Simultaneous Solution

To prevent the solution vector from drifting, a simultaneous adjustment including a relinearization of the design matrix must be performed periodically. This is accomplished in the program by actually maintaining two versions of the solution vector. One version remains constant throughout the updating procedure. This version changes only when the entire system is re-linearized through a simultaneous adjustment. The second version of the solution is used for updating. To obtain this solution, a back-substitution is performed into the current upper triangular matrix to derive parameter corrections. These corrections are applied to the constant solution vector to obtain the updated solution, which is echoed to the screen for the operators analysis.

With regard to on-line triangulation, the time at which a simultaneous solution should be performed is an important consideration. The most likely time is when the operator is involved with tasks unrelated to the sequential estimation. The best time for this would be as soon as possible after a new stereo-pair is introduced because the errors in approximate values are largest at this point. In our program, the operator has the ability to perform a simultaneous solution at his convenience.

TRIANGULATION METHODS AND RESULTS

As the calibration for the vision system cameras was not available at the time of writing, a simulated sequence of 5 stereo-pairs was mathematically designed to demonstrate the procedure necessary to sequentially triangulate an actual strip obtained by the mapping van.

Several assumptions were necessary in the design of this sequence. First, we assume that the van is traveling at approximately 55 miles per hour. This means that there should be about 24 meters between stereo-pairs. To match the stereo-vision system we constrain the base length to 1.83 meters and choose the interior and relative orientations of the cameras to be as close to reality as possible. These simulated orientations appear in Table 1. Assumptions are also made about the probable location of ground points which would appear in the images. By projecting these 3 dimensional coordinates into image space we obtain their image coordinates. All object coordinates and rotation angles refer to the vision coordinate system.

It must be emphasized that because this strip is simulated, the results obtained from the triangulation are too optimistic to expect from an actual sequence, and that the purpose here is simply to demonstrate how the van location can be sequentially tracked through a loss-of-lock period.

To begin the triangulation process, the datum is established by fixing the object coordinates of the camera perspective centers of the first stereo-pair along with the rotation angle of the left camera around the x axis (ω). To simplify matters, the coordinates of the left perspective center of the first stereo-pair are assumed to be at the origin of the vision system at (0,0,0) and the right perspective center at (1.83,0,0). At this time all points are measured in the first stereo-pair. At any time during the measurement procedure the current point solution and camera exterior orientation are available to the operator. In this sequence, a total of 22 points were measured. Eleven of these were measured in the first stereo-pair and a simultaneous adjustment was performed. For subsequent stereo-pairs, any tie points appearing previously were measured first and then a simultaneous adjustment was performed to reduce the errors in the orientation parameters of the new stereo-pair. After the adjustment, any additional points could be computed accurately by simply performing an update. This procedure is necessary because new point approximations are computed by intersection and rely on the orientation parameters of the current stereo-pair. Obviously, better orientation values yield more accurate approximate values for added points.

Due to the precision of the simulated strip, the exterior orientations of each stereo-pair did not change throughout the sequential adjustment. For this reason, only the values obtained by the final adjustment are presented. These appear in Table 2. Included are the coordinates of the camera perspective centers for each stereo-pair, the rotation angles (ω,ϕ,κ), and the standard deviations for each.

In Table 2, it can be seen that as the strip length increases, the precision decreases. This occurs because the strip is only controlled at the first stereo-pair and the geometry worsens as the strip becomes longer.

CONCLUSIONS AND RECOMMENDATIONS

A mobile mapping workstation has been developed at the **Center for Mapping** at **The Ohio State University.** The workstation integrates a GPS receiver, an inertial system, and a stereo-vision system for automatic recording of highway data. This article focuses on utilizing the stereo-vision system for solving the "loss-of-lock" problem which occurs when satellite signals cannot reach the GPS antenna due to obstructions.

The computer program developed for this research implements a sequential adjustment method known as Givens Transformations Without Square Roots to tie together a strip of stereo-pairs obtained by the vision system during loss-of-lock. This strip is formed in an on-line triangulation mode and is controlled by the GPS positions of the van before and after loss-of-lock. The perspective centers and orientations of the cameras are determined sequentially through the strip allowing the vehicle to be navigated through the loss-of-lock period.

Givens Transformations provide a method for solving least squares adjustments without forming the normal equations. Givens Transformations Without Square Roots requires significantly less operations than conventional Givens Transformations and thus has become of interest for problems requiring results in real-time, such as on-line triangulation.

Givens Transformations are advantageous in terms of numerical stability, efficiency, and storage requirements. Additionally, in an interactive mode, observations can be added and deleted and the updated solution vector and cofactor matrix can be obtained efficiently.

	Xo[m]	Yo[m]	Zo[m]	ω[rad]	φ[rad]	κ[rad]
STEREO-PAIR #1						
left	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
#std	0.00000	0.00000	0.00000	0.00000	0.00012	0.00005
right	1.83000	0.00000	0.00000	0.10000	0.10000	0.10000
#std	0.00000	0.00000	0.00000	0.00000	0.00012	0.00004
STEREO-PAIR #2						
left	1.50000	0.00000	-23.99000	0.00000	0.00000	0.00000
#std	0.00288	0.00073	0.00359	0.00002	0.00012	0.00008
right	3.33000	0.00000	-23.99000	0.10000	0.10000	0.10000
#std	0.00288	0.00087	0.00417	0.00003	0.00012	0.00008
STEREO-PAIR #3						
left	2.99000	1.00000	-48.00000	0.00000	0.00000	0.00000
#std	0.00557	0.00040	0.00460	0.00003	0.00012	0.00009
right	4.82000	1.00000	-47.99000	0.10000	0.10000	0.10000
#std	0.00557	0.00055	0.00467	0.00003	0.00012	0.00009
STEREO-PAIR #4						
left	4.49000	2.00000	-70.00000	0.00000	0.00000	0.00000
#std	0.00814	0.00054	0.00623	0.00003	0.00012	0.00009
right	6.32000	2.00000	-70.00000	0.10000	0.10000	0.10000
#std	0.00814	0.00063	0.00633	0.00003	0.00012	0.00009
STEREO-PAIR #5						
left	5.99000	1.00000	-90.00000	0.00000	0.00000	0.00000
#std	0.01047	0.00093	0.00840	0.00003	0.00012	0.00009
right	7.82000	1.00000	-90.00000	0.10000	0.10000	0.10000
#std	0.01047	0.00100	0.00850	0.00003	0.00012	0.00009

Table 2: Final Adjusted Orientation Parameters for Simulated Sequence

Large, sparse design matrices such as those frequently encountered in photogrammetry can be readily exploited by Givens Transformations. In the triangulation of a strip of several stereo-pairs with a large number of points, however, memory is always a concern. One possible solution to the problem would be to maintain a strip no longer than the number of stereo-pairs over which tie points have an influence. For example, if tie points from the first stereo-pair in a strip are no longer visible in the fourth stereo-pair, then the first could be dropped and the strip would now begin with the second. In this way no more than three stereo-pairs would be in memory at any time.

The approach depicted in this paper can be utilized in vehicles other than the van described here. For example, positioning from boats, trains, and airplanes is possible. The method described for tying together a strip of stereo-pairs is not limited to mapping applications. Any problem involving sequential imaging can potentially be approached in this manner. Examples would include biomedical stereo imaging and autonomous vehicle navigation which is becoming a very popular topic of research due to the rapid development of intelligent vehicle highway systems (IVHS).

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