

## AEROTRIANGULATION WITHOUT GROUND CONTROL

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Commission I

Abstract:

An integrated navigation system, which consists of an inertial measuring unit and a GPS-satellite receiver, is used to position the aerial camera at the moment of exposure. The incorporation of this information in a photogrammetric bundle adjustment makes aerotriangulation without ground control a distinct possibility for medium to small scale mapping projects. Results from a land-based prototype of such a system yielded coordinate accuracies around the 5 m level. However, these results contain considerable orbital errors and also reflect the early stage of receiver development. Thus significant future improvements are a possibility. It is anticipated that through the use of the improved system, exposure station coordinates will be determined with a standard error of 1 - 2 m, and following a full deployment of the GPS-satellites, scheduled for 1988, global use of such a system will be possible. This paper describes the system concept, outlines the necessary improvements, and presents the results of bundle adjustment simulation tests, which were based on the above stated exposure station positioning accuracies.

1. Introduction

Today, a standard tool for small and medium scale topographic mapping is photogrammetric block triangulation. In this method, known ground positions are used to transform the model coordinates, derived from a set of photogrammetric bundles or models, into geodetic coordinates. Transformations of this type should only be made between homogeneous sets of coordinates. While the homogeneity of the photogrammetric model coordinates is, in general, good, this cannot always be said of the given geodetic control. In areas with a mix of satellite derived positions and classical triangulation, distortions between the different sets of coordinates can occur. In some cases, they will be large enough to introduce systematic errors into the block adjustment. Furthermore, the establishment of ground control in unsurveyed areas is often costly and time consuming. In surveyed areas, restrictions on the flight pattern due to the existing control cannot always be avoided and loss of targets in densely populated areas is a common occurrence.

The idea to move the control from ground level to flight level is therefore attractive. It provides a set of homogeneous control coordinates at flying height, avoids targetting problems, and gives complete freedom in the choice of the flight pattern. The main advantage, however, is the capability for instantaneous positioning (Schwarz, 1983c). This means that the block triangulation could be repeated at any time with the same accuracy but without reference to a fixed set of control points. In each repetition, the camera positions at the instants of exposure would be determined

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and used as control. Control would therefore be instantaneous and would not require monumentation.

A system that provides the necessary positioning accuracy at flying altitude can be realized today by a combination of inertial and satellite techniques. In this combination, the satellite system provides absolute position information and thus repeatability, while the inertial system contributes highly accurate relative position information and thus homogeneity. Such an integrated positioning system has been tested on land and has produced very satisfactory results. Application to the airborne case and possible accuracy improvements will be discussed in this paper.

Some salient features of inertial technology which have been proposed earlier for application in photogrammetry can also be used with this system. They include the guidance capability of these systems for flight-line navigation (Bruland, 1982) as well as the use of attitude control as auxiliary information, see Reid et al (1980), Gibson and Masry (1981), and Blais and Chapman (1983). However, the emphasis in this paper is on the complete elimination of ground control rather than on these additional features. It thus is akin to techniques used in the Apollo lunar program where independence of ground control was achieved by deriving the positions of the exposure stations from satellite tracking data and the attitude angles from a stellar camera (Light, ed, 1980). Although the inertial reference is different in this case and much higher accuracy is aimed at, the principal idea is similar.

## 2. Principle of Operation and System Components

The principle of operation is shown in Figure 1.

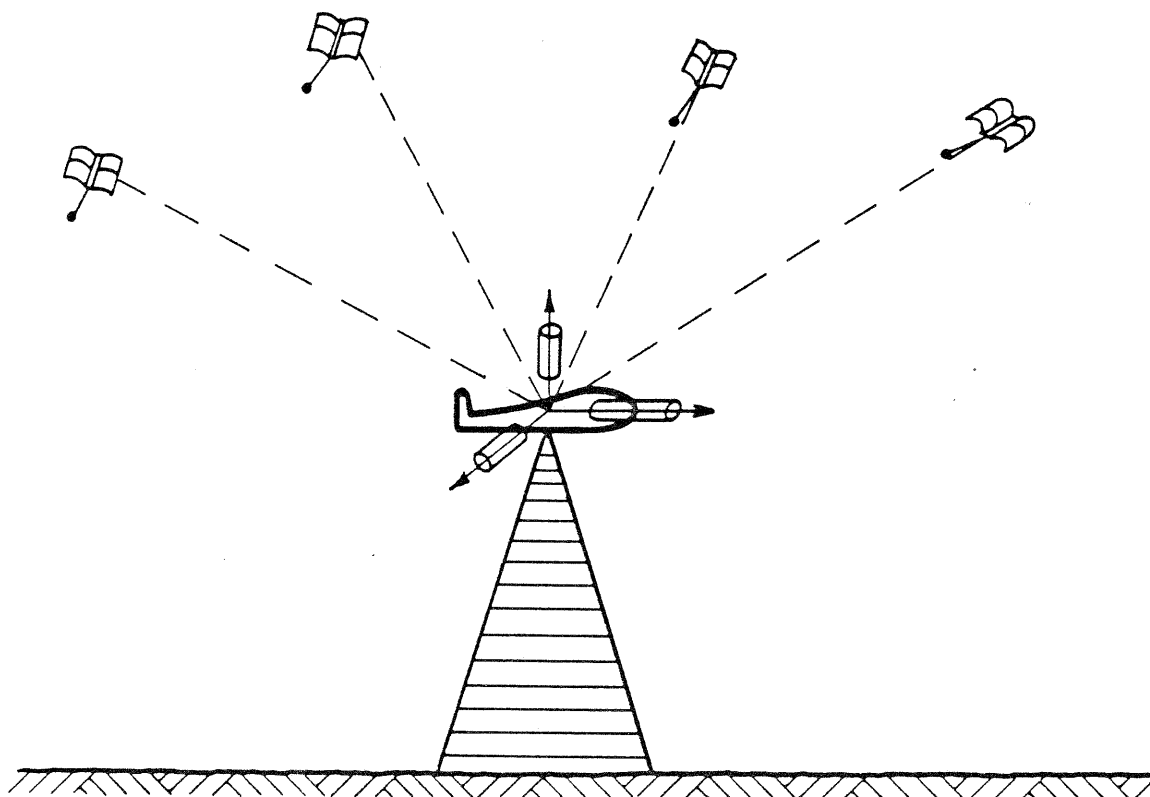


Figure 1 : Principle of the Proposed Method

An inertial navigation system, mounted in an aircraft, generates continuous position and velocity information. Due to systematic errors in the system the accuracy of the derived positions deteriorates as a function of time. The effects of these errors can be reduced by using external control measurements. They are provided by satellite ranges which are used as update information for a Kalman filter. The outputs of the Kalman filter are optimal real-time estimates of velocity, position, and attitude. To get optimal post-mission results, smoothing is required.

The main measuring unit in this configuration is the inertial navigation system. It consists of an attitude reference, an accelerometer triad, a real-time processor, and a clock. The attitude reference, instrumented by way of gyroscopes, establishes an internal orthogonal frame which keeps its orientation under motion. The accelerometer triad is aligned to this frame and measures accelerations in the three coordinate directions almost continuously, i.e. about every 20 milliseconds. The processor corrects the measurements for gravity field effects, Coriolis acceleration and system errors and then integrates the derived vehicle accelerations to obtain velocities and coordinate differences. The clock provides the independent variable in the equations of motion. Systems are mechanized either as strapdown or as stable platform systems. In the first case, the accelerometer is hard-mounted to the vehicle and experiences all vehicle rotations. In the second case, it is mounted on a gimballed platform and is essentially isolated from vehicle rotations. The stable platform systems are subdivided into the space stable system which keep their orientation fixed with respect to the distant galaxies ('inertial space'), and the local-level system that stays aligned to a local geodetic system. In the following, all references will be with respect to a local-level system because that system was used in the actual experiments.

Updating of the inertial navigation system is done by way of pseudorange to satellites of the NAVSTAR Global Positioning System (GPS). The NAVSTAR/GPS is a world-wide dynamic satellite positioning system which is scheduled to be fully deployed by the end of this decade. It will then provide instantaneous positioning with a standard error of 10 - 12 m by intersecting pseudorange measurements obtained from four satellites. In its final stage, it will consist of 18 usable satellites orbiting at an altitude of about 20 000 km with 12 hour periods. This will ensure that 4 to 7 satellites are available at any time anywhere on the globe. The proposed configuration in six orbital planes is shown in Figure 2. At present, four usable GPS-satellites are in orbit providing instantaneous positioning capability of about 2 hours in many parts of the world. Although updating of the inertial system is possible by individual ranges, sufficient accuracy can only be achieved when four satellites are available at the same time. This requirement can be reduced to three satellites if a Cesium clock can be integrated into the system. Characteristics of the planned system are well documented in 'Global Positioning System' (1980); a brief overview of the features important for this application are given in Wong and Schwarz (1983).

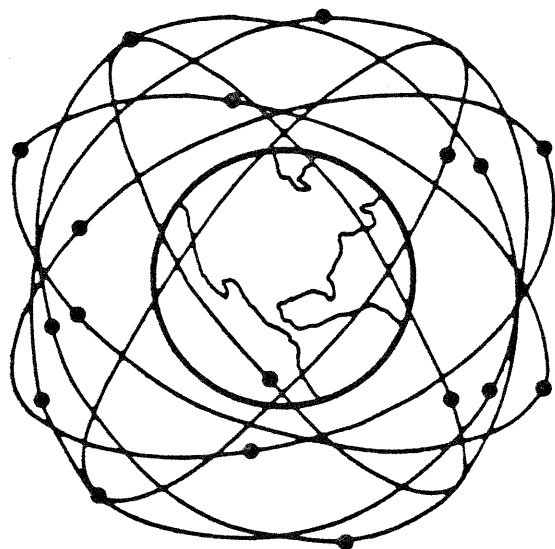


Figure 2 : NAVSTAR/GPS Configuration — Final Stage

### 3. Instantaneous Positioning Model

The general mechanization equations of an inertial survey system of local-level type are of the form

$$\begin{pmatrix} \dot{v}_E \\ \dot{v}_N \\ \dot{v}_U \end{pmatrix} = \begin{pmatrix} f_E \\ f_N \\ f_U \end{pmatrix} - \begin{pmatrix} 0 & -\sin\phi & \lambda\cos\phi \\ \lambda\sin\phi & 0 & \dot{\phi} \\ -\lambda\cos\phi & -\dot{\phi} & 0 \end{pmatrix} \begin{pmatrix} v_E \\ v_N \\ v_U \end{pmatrix} - \begin{pmatrix} \eta \\ \xi \\ \gamma + \Delta g \end{pmatrix} \quad (1)$$

where

- $v_E, v_N, v_U$  ... are the velocity components in east, north and upward direction
- $\dot{v}_E, \dot{v}_N, \dot{v}_U$  ... are the time derivatives of the velocity components
- $f_E, f_N, f_U$  ... are the specific force measurements in east, north, and upward direction
- $\lambda = \omega + \dot{\lambda}$  ... is the sum of the angular velocity of the earth ( $\omega$ ) and of the east component of vehicle motion ( $\dot{\lambda}$ )
- $\dot{\phi}$  ... is the angular velocity of the north component of vehicle motion
- $\phi$  ... is the geodetic latitude
- $\xi$  and  $\eta$  ... are the deflections of the vertical
- $\Delta g$  ... is the gravity anomaly
- $\gamma$  ... is normal gravity.

For a derivation of these equations, see e.g. Britting (1971) or Faure (1971). The firmware of the inertial system provides the continuous integration of equations (1). Thus, the output of the system is velocities and coordinate differences in latitude, longitude and height. This output is contaminated by a number of errors which are due to the dynamics of the system and are systematic in nature. They can be described by a system of differential equations of the form

$$\dot{\underline{x}} = \underline{F} \underline{x} + \underline{G} \underline{u} \quad (2a)$$

with initial conditions

$$\underline{x}(0) = \underline{x}_0 \quad (2b)$$

A linear, first-order system of this form is used as a standard model in many engineering applications and is usually called the state space model. Its components are

$\underline{x}(t)$  ... the state vector  
 $\underline{F}(t)$  ... the dynamics matrix  
 $\underline{G}(t)\underline{u}(t)$  ... the driving noise.

It gives the time rate of change of the major error sources in the presence of noise. Here, as in the following, underlined lower case letters are vectors, underlined upper case letters are matrices.

A typical state vector for an inertial navigation system is of the form

$$\underline{x} = \{\epsilon_E, \epsilon_N, \epsilon_U, \dot{\epsilon}_E, \dot{\epsilon}_N, \dot{\epsilon}_U, \delta\phi, \delta\lambda, \delta h, \delta\dot{\phi}, \delta\dot{\lambda}, \delta\dot{h}\}^T \quad (3)$$

where

$\epsilon_E, \epsilon_N, \epsilon_U$  ... are the platform attitude errors in east, north and upward direction  
 $\delta\phi, \delta\lambda, \delta h$  ... are errors in latitude, longitude, and height.

A dot above a quantity is again to be interpreted as a time rate of change, i.e.  $\{\dot{\epsilon}_E, \dot{\epsilon}_N, \dot{\epsilon}_U\}$  are platform drift rates,  $\{\delta\dot{\phi}, \delta\dot{\lambda}, \delta\dot{h}\}$  are velocity errors. For a more detailed discussion of individual error sources, see Schwarz (1983a) and Stieler and Winter (1982). In order to integrate the position information of the inertial navigation system with that of the NAVSTAR/GPS, the error model for the ranges should also be cast into the form (2). After applying corrections for the satellite clock offset, ionospheric and tropospheric correction, the following equation is obtained:

$$r = \{(x_s - x_p)^2 + (y_s - y_p)^2 + (z_s - z_p)^2\}^{1/2} - \delta t_r + \delta r(t - t_b) + e_r \quad (4)$$

where

$x_s, y_s, z_s$  ... are the satellite coordinates in the Average Terrestrial System

$x_p, y_p, z_p$	... are the receiver coordinates in the Average Terrestrial System
$\delta t_r$	... is the receiver clock bias at time $t_b$ , usually the start of the mission
$\delta r$	... is the clock drift rate
$e_r$	... contains modelling and random errors.

Of the parameters in equation (4),  $\delta t_r$  and  $\delta r$  can be modelled as state vector components. The term  $e_r$  can be represented by a low order Gauss-Markov process with a time dependent covariance function. However, statistical information on the behaviour of  $e_r$  is so scarce at the moment, that it is safer to consider it as a stationary random value. Thus, the complete state vector is of the form

$$\underline{x} = \{\epsilon_E, \epsilon_N, \epsilon_U, \dot{\epsilon}_E, \dot{\epsilon}_N, \dot{\epsilon}_U, \delta\phi, \delta\lambda, \delta h, \delta\dot{\phi}, \delta\dot{\lambda}, \delta\dot{h}, \delta t_r, \delta r\}^T \quad (5)$$

For a more detailed discussion of equation (4) and its determination from the actual satellite message, see van Dierendonck et al (1980) and Payne (1982). The solution of the system of differential equations (2) for the state vector (5) can be obtained in analytical form, see e.g. Wong (1982). It is of the general form

$$\underline{x}(t) = \underline{\Phi}(t, t_0) \underline{x}(t_0) + \underline{w}(t, t_0) \quad (6)$$

where

$$\begin{aligned} \underline{\Phi}(t, t_0) & \dots \text{ is the transition matrix} \\ \underline{w}(t, t_0) & \dots \text{ is the matrix superposition integral.} \end{aligned}$$

For the relation between equation (2) and (6), see Gelb (1974), and for some practical ways of determining  $\underline{\Phi}$ , see Schwarz (1983b). Equation (6) can be written as a recursive relation of the standard form used in Kalman filtering

$$\underline{x}_k = \underline{\Phi}_{k,k-1} \underline{x}_{k-1} + \underline{w}_{k,k-1} \quad (7)$$

To update this equation, the range equation (4) has to be expressed in terms of the state vector elements, i.e.

$$\underline{r}_k = \underline{A}_k \underline{x}_k + \underline{e}_k \quad (8a)$$

where  $A$  is obtained from (4)

$$\underline{A} = \{0, 0, 0, 0, 0, 0, \frac{\partial r}{\partial \phi}, \frac{\partial r}{\partial \lambda}, \frac{\partial r}{\partial h}, 0, 0, 0, 1, (t-t_b)\} \quad (8b)$$

The derivation of the partial derivatives is given in Wong and Schwarz (1982). Equations (7) and (8) with their corresponding covariance matrices are used to set up the process of Kalman filtering and optimal smoothing, see Gelb (1974).

#### 4. Accuracy of Exposure Station Positioning

A prototype of the integrated positioning system discussed in this paper has been tested on land (Wong et al, 1983) and in the offshore environment (Schwarz et al, 1984). It is expected that results for the airborne case will be very similar if the data can be dumped at a sufficiently fast rate. Since the flight pattern is very regular, the higher velocity will not affect the accuracy very much. A brief discussion of the results obtained to date will be given in this section and some suggestions for improvements will be made.

Figure 3 shows results of a simulation study (Wong and Schwarz, 1982). It is reproduced here to show the importance of the satellite receiver switching rate for the accuracy of the results. The abscissa shows the time interval needed to switch from one satellite to the next. This determines

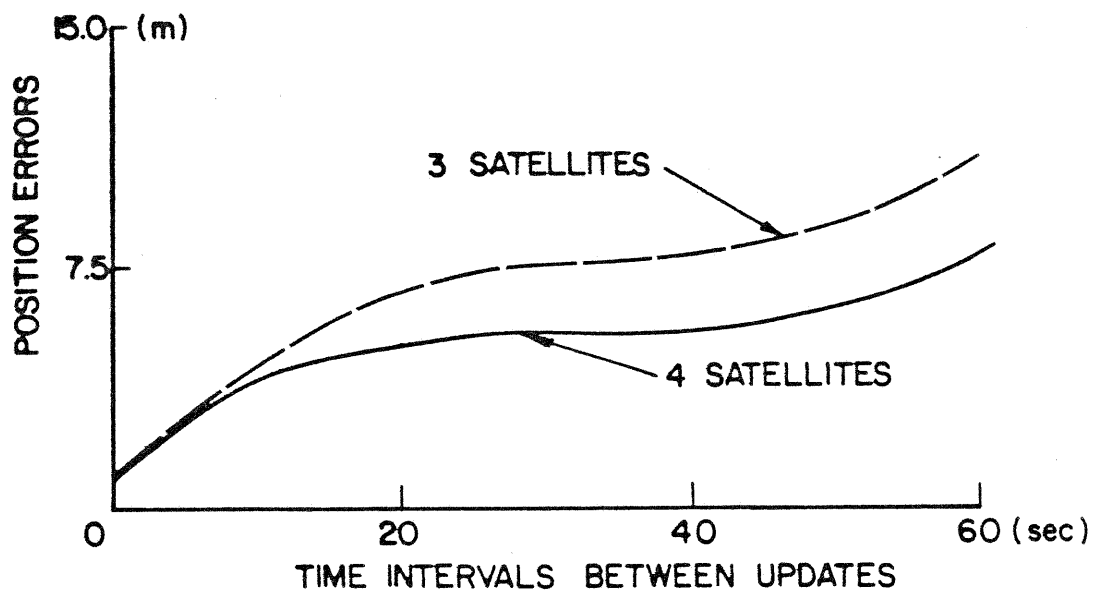


Figure 3 : Position Error as a Function of Update Interval

the time needed to obtain a new range measurement and thus the update interval in the Kalman filter approach. The time interval is different from one receiver to the next and varies presently between about 1 and 60 seconds. This means that the positioning accuracy to be expected from a slow switching receiver is in the range of 8 to 12 m ( $1\sigma$ ). By the same token, an accuracy of about 2 m can be expected from a fast switching receiver.

The simulation results were confirmed by tests on a baseline near Calgary (Wong et al, 1983). The integrated system consisted of a Ferranti Inertial Land Surveyor (FILS), an STI 5010 GPS receiver of Stanford Telecommunication Inc., an FTS 4050 cesium clock, and a Kalman filter package. It was developed as a joint project by NORTECH Canada and The University of Calgary with funding from the Natural Sciences and Engineering Research Council. The receiver was in this case of the slow switching variety (60 seconds). Thus, results in the 8 to 12 m range could be expected for a

single position fix without inertial input. A typical run along the 40 km L-shaped baseline is shown in Figure 4. The maximum errors at the given control points are always below 15 m. The standard errors for the best run are

$$\sigma_{\phi} = 6.9 \text{ m} \quad \sigma_{\lambda} = 3.3 \text{ m}$$

and for all runs

$$\sigma_{\phi} = 7.1 \text{ m} \quad \sigma_{\lambda} = 6.3 \text{ m} .$$

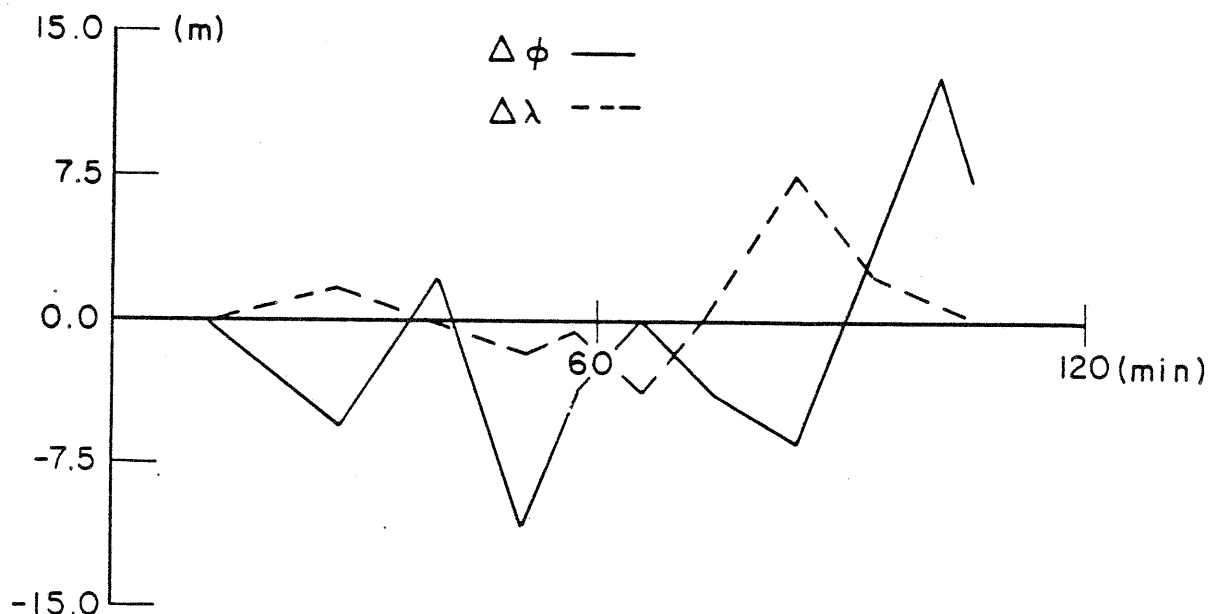


Figure 4 : Typical Positioning Error of Integrated INS-GPS Using Slow-Switching Receiver

These results are of the same order of magnitude as those obtained by Lachapelle et al (1983) for stationary point positioning. This indicates that the inertial system works as an almost perfect interpolator. It also shows that the range errors are at present not completely random. Since only a small number of tracking stations is currently in operation, orbital errors are the most likely explanation for the quasi-systematic effects. Once they can be reduced and a fast switching receiver employed, a dynamic positioning accuracy of 2 m should be achievable.

To improve this accuracy, the configuration shown in Figure 5 should be considered. Instead of using a receiver only in the aircraft, a second stationary receiver is used on the ground. This will minimize orbital and atmospheric errors if the moving receiver stays within a radius of about 100 km of the stationary receiver. Similarly, the timing errors will be reduced if the two receiver clocks are carefully calibrated before takeoff.



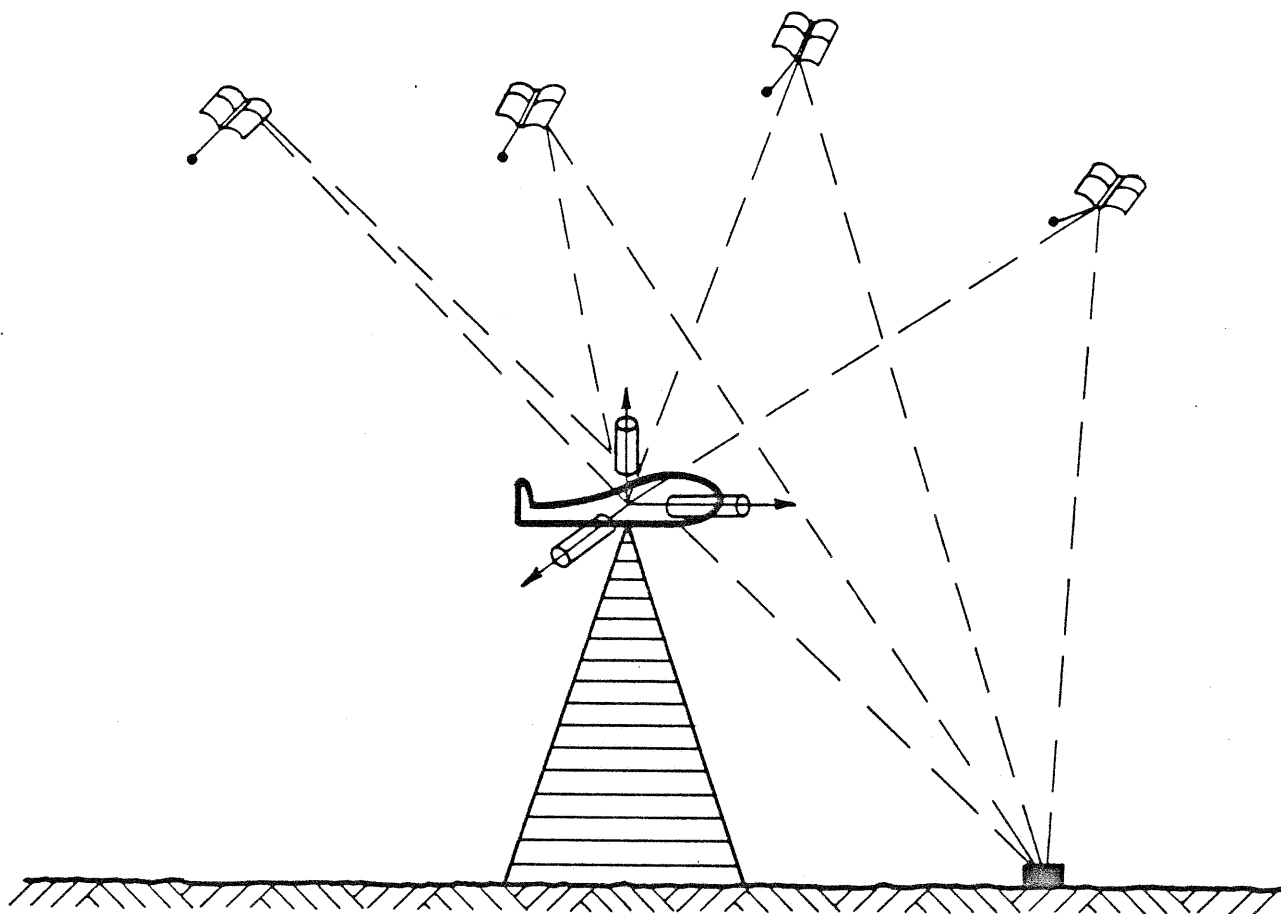


Figure 5 : Differential Mode of Proposed Method

In addition, the stationary receiver could be used as an inflight calibration mark which is photographed at regular time intervals to improve the post-mission smoothing. At a later stage, it may be possible to replace the inertial system by some simpler type of interpolator. This could lead to a considerable reduction in costs.

##### 5. Accuracy of Aerotriangulation Without Ground Control

Based on the considerations in the last section, it will be assumed that exposure station coordinates can be determined with an accuracy of 2 m using the present system, with improved orbit information and a fast switching receiver (Figure 1). A standard error of 1 m will be assumed for the two receiver configuration (Figure 5). Based on these assumptions, simulations with a bundle adjustment program were performed to study the potential of aerotriangulation without ground control. In all simulations only the coordinate information has been used.

Covariances for the attitude control have not been included in the block adjustments carried out since simulations have shown that such control, even of moderate accuracies, can be expected to have only a limited impact on the triangulation precision (Brown, 1968). To appreciate the limited influence of attitude control, one need only consider the photogrammetric

orientations implicit in a block adjustment. Firstly, the inherent strength of the relative orientation between photographs renders additional attitude data ineffective unless the accuracy of measurement of the  $\omega$ ,  $\phi$  and  $\kappa$  rotations is at a level of about 10 arc seconds or better. In the absolute orientation process the attitude in space of the block with respect to the ground coordinate datum is constrained through the GPS-inertial system supplied exposure station coordinates. Again, in this phase external attitude control cannot be expected to contribute significantly to point positioning accuracy. One conceivable block configuration that could benefit to some extent through the provision of additional camera attitude information, however, is the case where the sidelap is quite limited. Here, tilt ( $\omega$ ) control may enhance the leveling in individual strips, in the direction normal to the flightline.

A further point that should be noted is the effect of biases in interior orientation. In the absence of any ground control, biases in the principal distance and coordinates of the principal point will give rise to a scale error and a translation in the ground (x,y) coordinates respectively. The influence of interior orientation errors can, however, be readily corrected for so long as one ground control point is available, as in Figure 5.

Simulated data was produced for a 30 photo block containing three strips of equal length. A standard overlap of 60% and sidelap of 20% were adopted. A scale of 1:50 000 was achieved using an average flying height of 7 620 m and a 152.40 mm focal length camera (22.8 cm x 22.8 cm format). A total of 70 ground points were used in the simulation.

Photo coordinates and camera orientation and coordinates were all simulated. This data was then adjusted using several different control configurations. The first configuration utilized was a standard sparse control network. Four ground points located near the corners of the block were used as control in all three dimensions (X,Y,Z) and five other points, including one near the block centre, were used as auxiliary height control (Z only). The second configuration tested used only the four corner control points with no auxiliary height control. The final configuration tested used no ground control and relied on camera station coordinates alone.

A self-calibrating bundle adjustment approach was used in which accuracies of camera station coordinates can be included a priori in the adjustment. This extra information is introduced through the mathematical model used in the adjustment. The linearized form of the mathematical model of the bundle adjustment used is given by the matrix equation

$$\begin{pmatrix} V \\ \dot{V} \\ \ddot{V} \end{pmatrix} + \begin{pmatrix} \dot{A} & \ddot{A} \\ -I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} \dot{\delta} \\ \ddot{\delta} \end{pmatrix} = \begin{pmatrix} e \\ \dot{e} \\ \ddot{e} \end{pmatrix} \quad (9)$$

where

$\dot{\delta}, \ddot{\delta}$  represent the vectors of corrections to the exterior orientation elements and the object space coordinates;  
 $\dot{A}, \ddot{A}$  refer to the matrices of partial derivatives of the extended collinearity equations with respect to the exterior orientation elements and the object space coordinates;

$V, \dot{V}, \ddot{V}$  are the vectors of residuals for the image point coordinates, the exterior orientation elements, and the object space coordinates;  
 $e, \dot{e}, \ddot{e}$  indicate the discrepancy vectors; and  
 $I$  is the unit matrix.

In the form given by equation 9, all parameters are treated as observed or pseudo-observed quantities with a known a priori accuracy. In this case the a priori accuracy of the exposure stations is implemented by weighting through  $A$  of equation 9. For further discussion of the development and structure of this system the reader is referred to Fraser (1980) and Brown (1976).

Three adjustments were run for the first two control configurations, using different a priori standard errors as accuracy values for the camera station coordinates. These coordinates were first allowed to move freely by assigning standard errors of 200.00 m. Then standard errors of 2.00 m and 1.00 m respectively, were used. For the adjustments with no ground control, only the latter two accuracies were used. The results of these adjustments can be found in Table 1.

TABLE 1  
 Results of the Photogrammetric Bundle Adjustments  
 for the Simulated Data (all units are metres)

control configuration	camera station coordinates a priori standard error	standard errors of resulting ground coordinates (a)			mean standard errors of ground coordinates (b)
		$\sigma_X$	$\sigma_Y$	$\sigma_Z$	
Standard Sparse Control	200.00	0.591	0.627	1.612	1.101
	2.00	0.462	0.501	0.721	0.597
	1.00	0.386	0.429	0.626	0.513
Corner Control Only	200.00	0.662	0.740	2.441	1.567
	2.00	0.476	0.517	0.902	0.680
	1.00	0.399	0.442	0.714	0.552
No Ground Control	2.00	0.975	1.038	1.277	1.104
	1.00	0.700	0.763	0.962	0.816

(a) Values given are mean values for all 70 points including control points.

(b) Values given are mean values for all 70 points excluding control points.

Clearly the results obtained, especially with no ground control, are quite promising. Normal accuracy ratios of 1:10 000 for X and Y, and 1:8 000 for Z (accuracy ratio = standard error/average flying height) were obtained, or very nearly so, even with no ground control. Comparing the standard case (i.e. standard sparse control with no control on the camera stations) with the case with no ground control and 2.00 m standard error on camera

stations, mean standard errors are essentially equal. Standard errors on individual coordinates deteriorated by approximately 60% in X and Y but improved in Z by 20%. There were similar but better results using the 1.00 m standard error. There is also a more homogeneous solution in X, Y and Z using no ground control where the standard case is less accurate in Z than X and Y (significantly).

Maximum standard errors from the two cases discussed above reduced from 2.534 m in the standard case to 1.874 m and 1.469 m in the no ground control case with standard errors of 2.00 m and 1.00 m respectively (note - all maximum standard errors were in Z). Given these kinds of errors and those shown in Table 1, one could say that the ground point positioning accuracy obtained using no ground control is equal (or better) to the accuracy of the camera stations used as control.

Generally, based on the improvements planned in the positioning systems, aerotriangulation without ground control is certainly a possibility. The potential for a system such as this is obvious. Medium and small scale mapping projects could become not only more cost effective, but certainly much less time consuming.

## 6. Conclusions

Land-based tests with an integrated GPS-inertial system have shown that instantaneous positioning accuracies with a standard error of about 5 m ( $1\sigma$ ) can be achieved today. By using a fast switching receiver and improved orbital information the standard errors in X, Y, and Z can be reduced to about 2 m ( $1\sigma$ ). Further improvements are feasible by using a stationary as well as a moving receiver. Standard errors of 1 m ( $1\sigma$ ) seem to be possible.

The use of such a system in photogrammetric block triangulation has been studied. First results indicate that aerotriangulation without ground control is a distinct possibility for small and medium scale mapping. Accuracies of exposure stations are reflected almost directly in the positional accuracy of final ground coordinates. Using a standard error of 2 m ( $1\sigma$ ) for the exposure station coordinates and no ground control gives results that are comparable to the standard case of sparse ground control and no weight on exposure station coordinates. In addition, coordinate accuracies seem to be more homogeneous for the adjustment without ground control.

## Acknowledgement

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