

On The Stability of Lenses for Aerial Mapping Photography  
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#### ABSTRACT

About 200 calibrations for 28 different lens/filter combinations including cameras of the types Wild RC8, Wild RC10 and Zeiss RMK, and lenses of the types Wild Aviogon, Wild Universal Aviogon (for RC8 and RC10), Wild Universal Aviogon I, Wild Universal Aviogon II, Wild Super Aviogon II, Zeiss Pleogon A, Zeiss Pleogon A2, Zeiss Pleogon A4 and Zeiss Super Pleogon A have been recomputed to obtain as many data as possible suitable for an evaluation of the stability of aerial cameras.

#### INTRODUCTION

Most aerial cameras are believed to be rather stable. This suggests that periods between recalibrations of cameras could be rather long if nothing happens to the camera which could influence its overall geometrical-optical characteristics. The Canadian Interdepartmental Committee on Air Surveys has in the past insisted that a camera to be used on a contract to procure aerial photography for Federal topographic mapping activities must have been calibrated within the last 12 months preceeding the photography. This requirement has provided series of recalibrations suitable for a stability investigation.

The first calibration of an aerial camera to be used for mapping purposes was carried out by a Canadian Federal institution in 1922 using a visual method. Since 1931, calibrations of aerial mapping cameras were carried out at the National Research Council laboratories using visual methods until 1955, and photographic procedures since. The present calibrator [Carman and Brown 1978] was first taken into service in 1967 and modified in 1973.

Between December 1973 and August 1981 inclusive 632 photographic plates were taken to calibrate different lens/filter combinations with 121 different lenses of 21 lens types, and with 204 different lens/filter combinations. Of these, 198 plates for 28 lens/filter combination were measured and processed for this report. These 28 series of calibrations include each at least five calibrations of a lens/filter combination within the  $7\frac{1}{2}$  year time period covered. The determined data include the location of the fiducial marks, the principal point of autocollimation and the point of best symmetry, the calibrated focal length, the rotationally symmetrical lens distortion and the decentering distortion.

This paper only gives an overview over the obtained results. A detailed analysis of the data will take place at a later date elsewhere.

#### INPUT DATA

All plates were measured on a Zeiss PSK1 comparator used as monocomparator, and two sets of measurements were taken using a reversed point order for the second set. The reading differences were determined and points with differences exceeding  $10\mu\text{m}$  remeasured. Since the target to be measured is a cross oriented at  $45^\circ$  to the comparator axes, pointing was somewhat difficult,

and about two points per plate were remeasured after the differences check.

Each plate contains images from three exposures: the fiducial marks, collimator images along one diagonal and collimator images along the second diagonal. The plate is shifted between second and third exposures to avoid a double exposure of the central collimator image, and the second and third exposures are adjusted to be taken after an exact rotation of  $90^\circ$ . Collimators are located at  $45^\circ/16$  intervals. All visible collimator images were used in this investigation for all plates taken after October 1974, and only the images of the collimators located at  $i.45^\circ/16$  with  $i = 0, 2, 4, 6, 8, 10, 12, 14, 15, 16, 18$  and  $20$  for plates taken between December 1973 and October 1974. These locations are those used routinely and calibrated data for the collimator locations were until October 1974 only derived for these collimators.

The collimator locations are recalibrated in approximately 12-month intervals using a procedure which bisects angles sequentially, and calibration data are available for December 1973, October 1974, September 1975, August 1976, September 1977, December 1978, March 1980, November 1980 and November 1981. Later calibration data are not of interest here. Typical variations during the covered time period do not exceed 2" between any two of the calibrations, although larger changes in position occurred for at least two of the 43 collimators.

#### FIDUCIAL MARKS

All measured coordinates were transformed into an ideal fiducial mark coordinate system defined with the origin at the centre, and fiducial mark locations of  $(+106, +106)$ ,  $(+106, -106)$ ,  $(-106, -106)$  and  $(-106, +106)$  for Wild cameras and  $(+130, 0)$ ,  $(0, +130)$ ,  $(-130, 0)$  and  $(0, -130)$  for Zeiss Oberkochen cameras (all coordinates in mm). The residuals obtained after linear conformal transformations for each set of calibrations are located within a range of  $5\mu\text{m}$  from the set average, with few exceptions. These exceptions also show larger scale changes; hence, the scale variations can be used as an indicator for the stability of the fiducial marks. Fig. 1 presents the scale changes within each series of calibrations plotted against time of calibration. Vertical lines represent June and December of each year, horizontal lines are spaced at scale change intervals of 0.00005. The scale values within each of set of calibrations for the 19 series of Wild camera data vary within a range of 0.00006, and six of nine series for Zeiss Oberkochen cameras data fall also within this range.

The calibration of series 20 carried out in May 1980 shows a scale change of 0.00092 compared to the preceding calibration of January 1979. An analysis of the distances between the fiducial marks indicates that this change was caused by the change of one fiducial mark. The angle of intersection between opposite fiducial marks changed enough to reach the limit of deviation from  $90^\circ$  allowed for in the Canadian specification; the camera owner was informed about this fact. When the camera was recalibrated in August 1981 additional changes in this and at least one other fiducial mark had taken place, and the camera failed to meet the requirements of the specifications for a camera for aerial photography for Canadian Federal topographic mapping. The camera has since been repaired and now meets again the specification.

Series 21 shows two scale changes: the first of 0.00052 between the calibration of April 1974 and May 1975 was caused by the movement of one fiducial mark. This change did not affect the classification of the camera. A second fiducial mark moved between December 1977 and June 1980. The camera

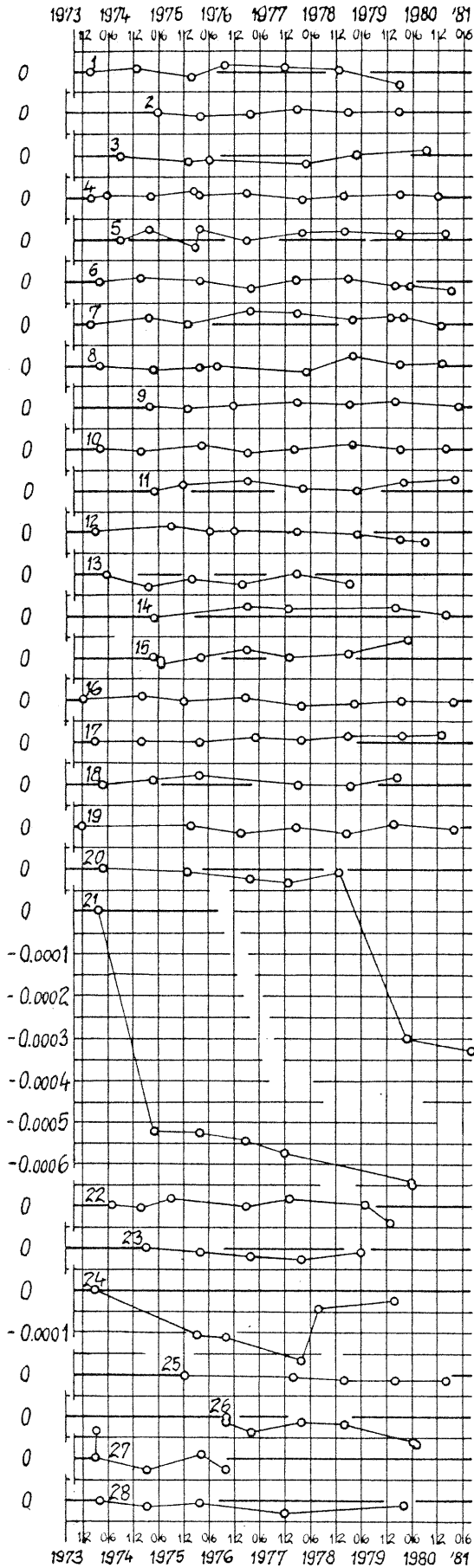


Fig. 1 (see text)

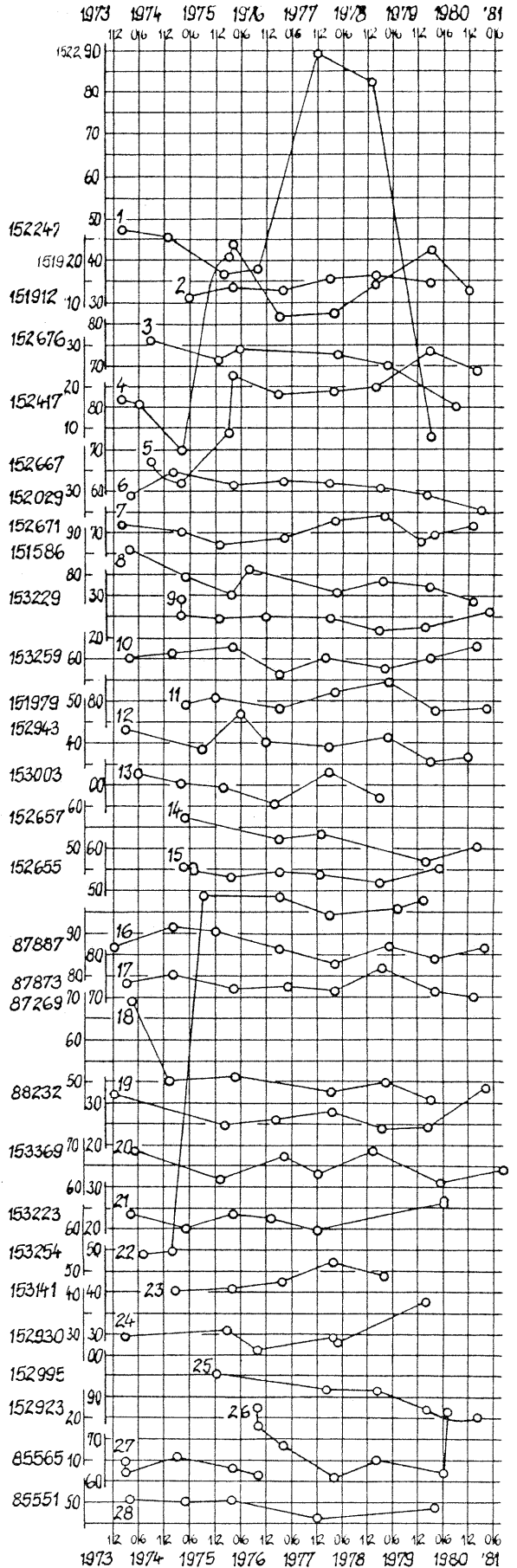


Fig. 2 (see text)

now failed to meet the requirements of the specification for a camera for Canadian Federal topographic mapping aerial photography. A recalibration was carried out the following day which confirmed the preceding calibration.

Series 24 also indicates scale changes which can be shown to be the result of changes in fiducial mark locations. The calibration of April 1978 showed the camera near the limits permitted in the specification in regard to the angle of intersection of the lines joining opposite fiducial marks. The fiducial marks were reset and the recalibration of May 1978 shows the camera to be well within specification.

#### COMPUTATIONAL CALIBRATION PROCEDURE

The employed computational procedure [Ziemann 1982] is based on a projective transformation extended with terms to correct for image distortion. The procedure was carried out in five steps as follows:

1) Determination of calibrated focal length  $f_c$   
 In this first step only three uncorrelated unknowns are used,  $f_c$  and a rotation around a vertical axis for each of the two exposures. Since the emulsion is set-up to be perpendicular to the central collimator, the other rotations which are part of the exterior orientation of a camera, can be considered equals to zero for both exposures. Also, since the camera is mounted above the camera calibrator such that the apex for the axes of the collimators is located in the entrance pupil of the lens, the position of the camera at the time of exposure is known. Finally, the rotationally symmetrical lens distortion is corrected as known for the lens type under calibration using in this instance a six-term polynomial. The standard deviation for  $f_c$  varies for the 198 calibrations between 1.12 and 2.12  $\mu\text{m}$  for the wide-angle lenses, and between 0.71 and 1.17  $\mu\text{m}$  for the super-wide angle lenses.

2) Preliminary Determination of Lens Distortion  
 During this step, an increasing number of lens distortion coefficients are introduced as unknowns. The stationally symmetrical lens distortion

$$\Delta x_s = x(s_0 + s_2 r^2 + s_4 r^4 + s_6 r^6 + s_8 r^8 + s_{10} r^{10}), \quad \Delta y_s = y(s_0 + \dots + s_{10} r^{10})$$

is known for each lens type. It was decided that the coefficients  $s_0$ ,  $s_8$  and  $s_{10}$  would be held throughout. The decentring distortion

$$\Delta x_s = x \left( \frac{3x^2 + y^2}{xr^2} \sin \tau + \frac{2y}{r^2} \cos \tau \right) (d_0 + d_2 r^2 + d_4 r^4)$$

$$\Delta y_s = y \left( \frac{2x}{r^2} \sin \tau + \frac{x^2 + 3y^2}{yr^2} \cos \tau \right) (d_0 + d_2 r^2 + d_4 r^4)$$

is initially assumed to be zero. Since  $\tau$  can only be determined when not all  $d_i$  are zero, the coefficient were introduced as follows:

- a)  $s_2, s_4, d_0$
- b)  $s_2, s_4, s_6, \tau, d_0, d_2$
- c)  $s_2, s_4, s_6, \tau, d_0, d_2, d_4$

The gradual introduction of additional unknown lens distortion coefficients also helps to overcome the problem created by the high correlation of the



Each iteration step but step 3 were repeated until the absolute value for the corrections for each of the unknowns was smaller than the respective standard deviation.

#### PRINCIPAL POINT OF AUTOCOLLIMATION

The principal point of autocollimation is, as a result of the set-up during the exposures of the plate, identical with the image of the central collimator [Carman and Brown 1978]. The measured coordinates for the first exposure of this point are used to relate this point to the fiducial marks, the measurements for the second and first images of the point to shift all images of the second exposure parallelly such that first and second exposure of the central collimator coincide.

The principal point of autocollimation was related to the fiducial centre after linear conformal transformation of the measured fiducial mark coordinates to coordinates defining the ideal fiducial mark locations by application of the transformation coefficients. The values for the different calibrations belonging to one set vary more than could be expected as a result of random measuring errors. Although extreme care is taken during the various steps preceding the exposure of a calibration plate, the possibility of set-up differences as cause for the observed variations of in some cases more than  $25\mu\text{m}/\text{coordinate}$  cannot be ruled out. It may be possible that these variations will become smaller, if the coordinates of the principal point of autocollimation are introduced as unknowns either in step 3 or in a step combining steps 2 and 3.

#### POINT OF BEST SYMMETRY

The point of best symmetry changes its location for the different calibrations of the same camera in a way similar to the principal point of autocollimation. However, the differences between the coordinates of these two points also change. The amount of the changes frequently exceeds the standard deviation for the coordinates of the point of best symmetry which is approximately  $4\mu\text{m}$  for the wide-angle cameras and approximately  $1.5\mu\text{m}$  for the super-wide-angle cameras.

#### CALIBRATED FOCAL LENGTH

The calibrated focal length is a multiplication factor which relates measured image coordinates to the given object-space geometry. It is determined in step 1 of the calibration procedure and retained unchanged thereafter, with a standard deviations between  $1.1$  and  $2.2\mu\text{m}$  for wide-angle cameras and  $0.7$  and  $1.2\mu\text{m}$  for super-wide-angle cameras. The lower value for super-wide-angle cameras is the result of the larger number of available points per exposure, 43 versus 29 for Wild wide-angle cameras and 31 for Zeiss wide-angle cameras.

Although the image coordinates were corrected for each camera by the rotationally symmetrical lens distortion of the camera type, the calibrated focal lengths between the different lenses of the same type vary notably:

Wild Aviogon lenses	between	151.91 mm	and	152.68 mm	for	3 lenses
Wild UAg (RC8)	"	" 152.03 "	"	" 152.69 "	" 4 "	"
Wild UAg (RC10)	"	" 151.57 "	"	" 153.26 "	" 4 "	"
Wild UAg II	"	" 152.65 "	"	" 153.00 "	" 3 "	"
Wild SAg II	"	" 87.25 "	"	" 88.23 "	" 4 "	"
Zeiss Pleogon A2	"	" 152.93 "	"	" 153.37 "	" 5 "	"

Figure 2 presents the focal lengths for all 28 series of calibrations. Vertical lines represent June and December of each year, horizontal lines a change in calibrated focal lengths of  $5\mu\text{m}$ . The changes within each series are in general limited to an envelope  $10\mu\text{m}$  or less wide; however, six of the series show changes of more than  $10\mu\text{m}$  between calibrations. Series 1, 4 and 5 all are for older Wild Aviogon or Universal Aviogon lenses, series 18 is for a Super Aviogon II lens, and series 22 and 26 are for Pleogon A2 lenses. The large change in series 22 was observed after a complete overhaul of the camera at the factory.

#### ROTATIONALLY SYMMETRICAL LENS DISTORTION

The Canadian Specification for Aerial Survey Photography specifies that the rotationally symmetrical lens distortion shall not deviate from the reference lens distortion for the lens type not more than  $5\mu\text{m}$  for field angles less than or equals to  $42.2^\circ$ , not more than  $10\mu\text{m}$  for field angles greater than  $42.2^\circ$  but less than or equals to  $53.5^\circ$ , and not more than  $15\mu\text{m}$  for field angles greater than  $53.5^\circ$ . For wide-angle cameras,  $42.2^\circ$  represents a radial distance of approximately  $139\text{mm}$ , and  $53.5^\circ$  falls outside the format area. Inspection of the obtained polynomials for the 159 wide-angle lens calibration shows that 15 exceed the set limits: 2 at  $131\text{mm}$ , 1 at  $133\text{mm}$ , 3 at  $134\text{mm}$ , 1 at  $135\text{mm}$ , 2 at  $137\text{mm}$  and 1 at  $138\text{mm}$ . All but one of these cases could be brought into the specified limit by a change in the calibrated focal length. Present routine calibration procedures would do this since the calibrated focal length is determined such that the maximum deviation from the standard values would be the same on both sides of the reference data. The data on which this report is based were determined using a least squares adjustment procedure throughout. The standard deviation for the rotationally-symmetrical lens distortion is obtained from

$$s_{\Delta r}^2 = P \cdot D \cdot C \cdot D \cdot P^T$$

where P is the row vector of the partial derivatives, i.e.  $[r^3 \ r^5 \ r^7]$ , D is a diagonal matrix with the standard deviations  $s_{s_2}$ ,  $s_{s_4}$  and  $s_{s_6}$ , and C is the correlation matrix. This equation yields for  $s_{\Delta r}$  the following values: at  $r = 2\text{cm}$   $0.1\mu\text{m}$ , at  $r = 4\text{cm}$   $0.4\mu\text{m}$ , at  $r = 6\text{cm}$   $0.8\mu\text{m}$ , at  $r = 8\text{cm}$   $1.1\mu\text{m}$ , at  $r = 10\text{cm}$   $1.1\mu\text{m}$ , at  $r = 12\text{cm}$   $1.6\mu\text{m}$  at  $r = 14\text{cm}$   $2.6\mu\text{m}$  and at  $r = 15\text{cm}$   $7.9\mu\text{m}$ . Given these values and the standard deviation for the calibrated focal length of between 1 and  $2\mu\text{m}$ , an arbitrary change in the calibrated focal length to balance maximum deviations does not appear to be justified, and another criterion for the goodness of fit between actual and reference lens distortion curves appears desirable.

The above-mentioned limits for deviations from the reference lens distortion are for super-wide-angle cameras valid for radial distances to approximately  $79\mu\text{m}$ , from  $79$  to approximately  $118\mu\text{m}$  and greater than  $118\mu\text{m}$ . Only one of 39 determined lens distortion curves fails to stay within the limiting values but could be made to stay within with a change in the calibrated focal length. The standard deviations for super-wide-angle and wide-angle lens distortion polynomials are nearly identical.

#### DECENTRING DISTORTION

The decentring distortion data obtained for calibrations within a series indicate in most cases a common tendency for the decentring distortion profile defined by the polynomial, and for the orientation angle  $\tau$  indicating the direction of maximum radial asymmetry. However, variations in the coefficient

$d_0$  indicate that the calibrated focal length  $f_c$  determined in step 1 may not be best. However, a change of  $f_c$  would also require a change in the term  $s_0$  which defines the relationship between equivalent and calibrated focal lengths.

The decentring distortion results point to possible insufficiencies in the described iteration procedure and to a need to agree on the treatment of decentring distortion data. The standard deviations for the polynomial defining the decentring profile were determined similar to those for the polynomial defining the rotationally symmetrical lens distortion. The following values were obtained: at  $r = 2\text{cm}$   $0.3\mu\text{m}$  at  $r = 4\text{cm}$   $0.3\mu\text{m}$ , at  $r = 6\text{cm}$   $0.4\mu\text{m}$ , at  $r = 8\text{cm}$   $0.4\mu\text{m}$ , at  $r = 10\text{cm}$   $0.5\mu\text{m}$ , at  $r = 12\text{cm}$   $0.6\mu\text{m}$ , at  $r = 14\text{cm}$   $1.1\mu\text{m}$  and at  $r = 15\text{cm}$   $2.1\mu\text{m}$ . Since the decentring distortion in practically all cases exceeds these standard deviations, it must be concluded that decentring distortion is not only present, but also, that it changes by amounts in excess of the standard deviation between calibrations.

Data were available for the two diagonals only. It therefore remains to determine whether an improvement in the determination of the decentring distortion could be achieved with the addition of off-diagonal points.

#### CONCLUSION

The reported data support the belief that aerial lenses are fairly stable. However, changes do occur. No attempt has been made at this time to determine the impact of these changes on photogrammetric results. The investigation shows also that certain aspects in camera calibration lack a convention in regard to definition; this is in particular so far the reporting of the decentring distortion.

#### ACKNOWLEDGEMENT

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