# PARALLELITY OF THE STEREOMETRIC CAMERA BASE TO THE DATUM PLANE

Dr. Oğuz MÜFTÜOĞLU
Faculty of Civil Engineering
Istanbul Technical University
Maslak - Istanbul - TURKEY

Commission V

ABSTRACT

*In an façate elevation drawing, stereometric camera base must be parallel to the datum plane at the time of exposure. If they are not parallel to each other, the scale will change on the image plane and also orthogonal and affinity errors will occur. In order to reduce the effect of these errors, an auxiliary apparatus was developed, which consist mainly of a double-pentagonal prism and of a base bar. Two miniature rods are set up at the ends of the base bar, which are parallel to each other and normal to the base bar. The third rod is held at any point on the line which is perpendicular to the datum plane and passes through the camera station. The image of this rod must fit into the image of the miniature rods of the apparatus in the prism.*

## 1. INTRODUCTION

The normal case of terrestrial photogrammetry is realized most conveniently by using stereometric cameras in which the base is fixed.

The advantage of the stereometric cameras are fixed relative orientation between the two cameras. The resulting photographs can be measured quite precisely by means of a parallax bar, or else in a stereoscopic plotting instruments. However, scale and displacement errors must be generated due to the focal plane not being parallel to the datum plane. The WILD Autograph A40 and the ZEISS Terragraph are both designed to plot from photographs taken in the normal case. The plate holders have no rotational motions, although equal elevation or depression angles at the two ends of the base can be accommodated.

An axuiliary apparatus was developed at this study and also new displacement equations were presented. A useful working document can be provided using this auxiliary aparatus.

## 2. ERRORS DUE TO ROTATION BY $\phi$ OF THE OBJECT PLANE

### 2.1. Errors of the Single Image Due to Rotation by $\phi$ of the Object Plane

The displacements in the image due to the position of focal plane are dependent upon the rotation of the object plane with respect to the datum plane from which the distance to the given camera station. Evidently, it can be seen in Figure 1.

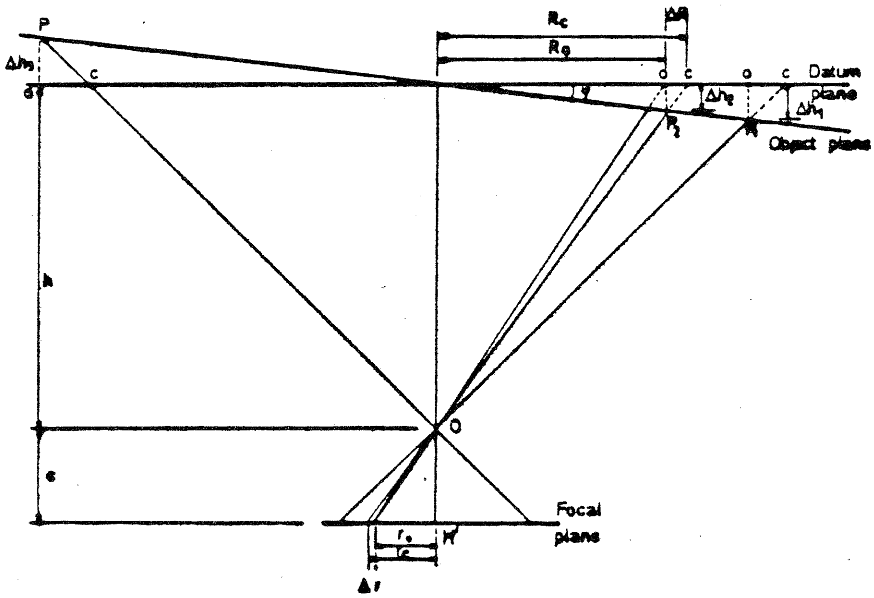


Figure 1. Section through the camera axis during horizontal photograph from rotated object plane

The following relations can be derived from this figure (these equations were developed due to elevation differences of the ground by B. Hallert (1960))

$$\Delta R = \frac{\Delta h \ R_c}{h} \qquad (2.1.1)$$

$$\Delta r = \frac{\Delta R \ c}{h} \qquad (2.1.2)$$

and accordingly

$$\Delta r = \frac{\Delta h \ R_c \ c}{h^2} \qquad (2.1.3)$$

since

$$R_c = \frac{h \ r_c}{h} \qquad (2.1.4)$$

$$\Delta r = \frac{\Delta h \quad r_c}{h} \qquad (2.1.5)$$

In addition to this relations, an equation of expected $\phi$ rotation for a single image was offered by R. Meyer (1970). According to the

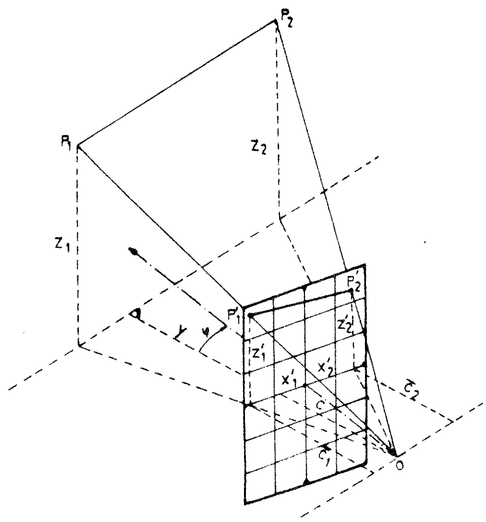

Figure 2. The geometrical relation between object and picture. R. Meyer (1970)

Figure 2, this expected error can be calculated with the following equations :

$$d\phi = \frac{0{,}5\ c\ \rho}{e'\ z'} \qquad (2.1.6)$$

where

$$e' = x_1' - x_2'$$

$$z' = (z_1' + z_2')/2$$

2.2. Errors of the Stereo Image Due to Rotation by $\phi$ of the Object Plane

In the analytical approach or analogue plotting, measurements of an image point are normally done according to the image coordinate system. $\Delta X$, $\Delta Y$ and $\Delta Z$ displacements can be expressed as a function of $\phi$ rotation, that is angle between the object plane and the datum plane.

X, Y and Z are the object-space coordinates of point P. These can be calculated in the following manner:

$$X_p = \frac{b}{P_p}\ x_p' \quad ; \quad Y_p = \frac{b}{P_p}\ c \quad ; \quad Z_p = \frac{h}{P_p}\ z_p' \qquad (2.2.1)$$

where

$$P_p = (x_p - x_p')$$

Object-space coordinates of point P' can also be calculated according to the normal case of stereophotogrammetry and these equations are in the following manner :

$$X_{p'} = \frac{b}{P_{p'}} x'_{p'} \quad ; \quad Y_{p'} = \frac{b}{P_{p'}} c \quad ; \quad Z_{p'} = \frac{b}{P_{p'}} z'_{p'} \qquad (2.2.2)$$

P is placed on the object plane and its rotated position on the datum plane is P' . The datum plane is parallel to the focal planes, consequently to the base line.

$\phi$ rotation will cause the displacements in the three dimension. These are expressed as,

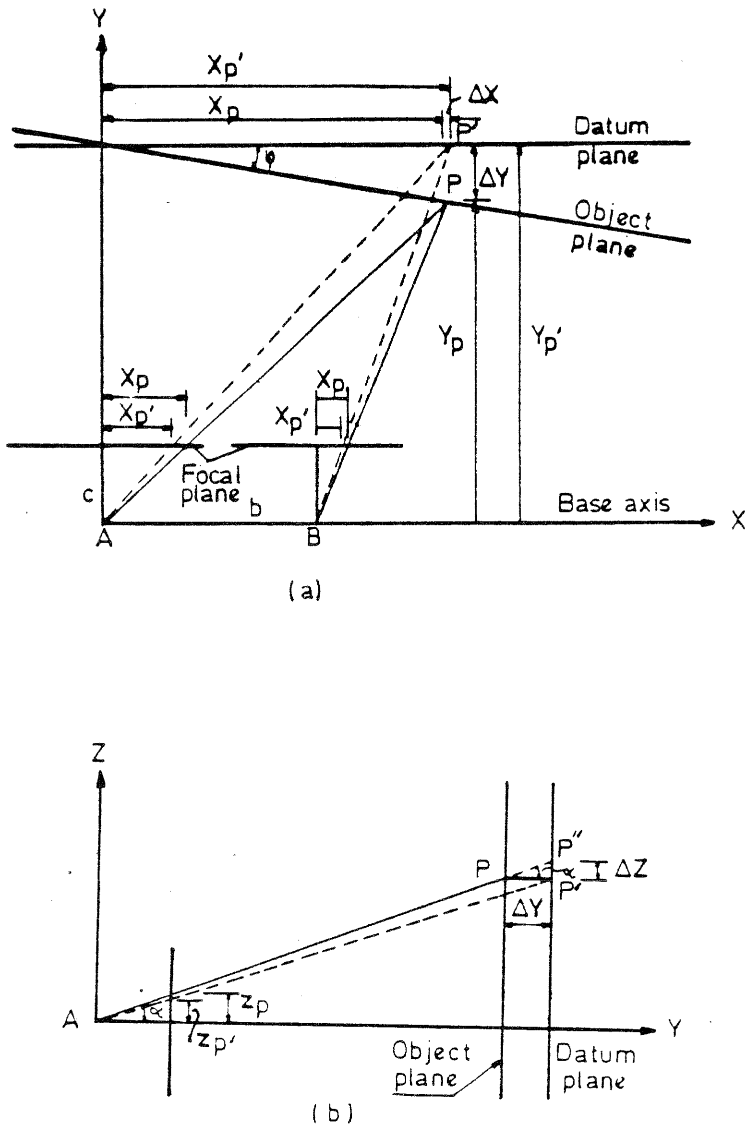


Figure 3. (a-b) The displacements of object point due to rotation of the object plane according to the datum plane

$$\Delta X = X_p(\sec\phi - 1) \quad (2.2.3)$$

$$\Delta Y = X_p \tan\phi \quad (2.2.4)$$

$$\Delta Z = \frac{Z_p}{c} X_p \tan\phi \quad (2.2.5)$$

A graphical output is plotted on a two dimension plane. The displacement of $\Delta Y$ is not effect graphical output through the Y-axis, but only $\Delta X$ and $\Delta Z$ displacements will be generated on the graphical output. These two displacements from the true position of point P will be seen as reduced length by drawing scale. The maximum standard of accuracy in these two dimensions can be expressed with the drawing scale and the standard 0.2 mm line width, that is

$$\max\Delta X \text{ (or } \Delta Z) = 0.2 \cdot m_p \quad (2.2.6)$$

From the equations (2.2.3) and (2.2.6), a maximum limit of the plane rotation can be calculated in the following manner :

$$\max\phi = \arccos \frac{X_p}{X_p - \max\Delta X} \quad (2.2.7)$$

## 3. AN AUXILIARY APPARATUS FOR THE PARALLEL ORIENTATION OF STEREOMETRIC CAMERA BASE

If the focal plane were parallel to the datum plane, analogue plotting can be done in the minimum error limits with the pair of photographs which are obtained in adequate position of the normal case of stereophotogrammetry.

In stereometric cameras which are manufactured according to the normal case of stereophotogrammetry, the base axis must accurately be parallel to the focal plane. Therefore, the base axis can be used instead of the focal plane.

Lets assume q is the normal plane that is perpendicular to the datum plane and passes through the base axis, 00' is intersection line of the normal plane and the datum plane (Figure 4). In the practical applications, if the 00' intersection line and the base axis are accurately made clear and parallel to each other, consequently focal planes are made sure parallel to the datum plane.

An auxiliary apparatus was developed in order to find base axis of stereometric camera and to made parallel to the datum plane. This apparatus mainly consist of a double-pentagonal prism and of a base bar (Figure 5). Also two miniature rods are set up at the ends of the base bar, which are parallel to each other and normal to the bare bar. Double-pentagonal prism is also set up between the miniature rods, by attaching on the middle of base bar. Two reflections of miniature rods look like one rod in the prism to the eye of observer which means the rods are on

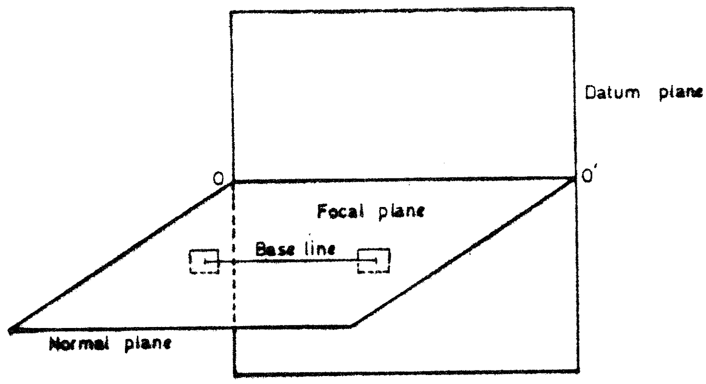


Figure 4. Geometrical relations between base line, focal plane, datum plane and normal plane.

the base line. The third rod is held at any point on the line which is perpendicular to the datum plane and passes through the camera station.

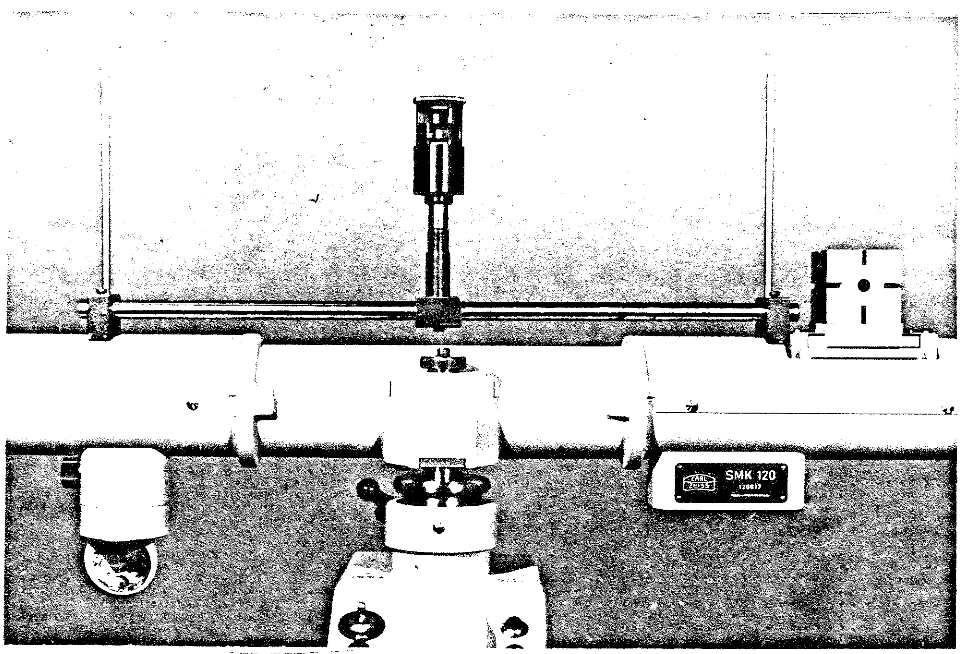


Figure 5. Auxiliary apparatus for the parallel orientation of stereometric camera base.

The image of this rod must fit into the reflections of the miniature rods of the apparatus in the prism (Figure 6). In order to secure the fitting of those images, the stereometric camera can be turned around its vertical axis. Thus, parallelity is obtain at the end of this procedure. This apparatus is easily used with a little practice.

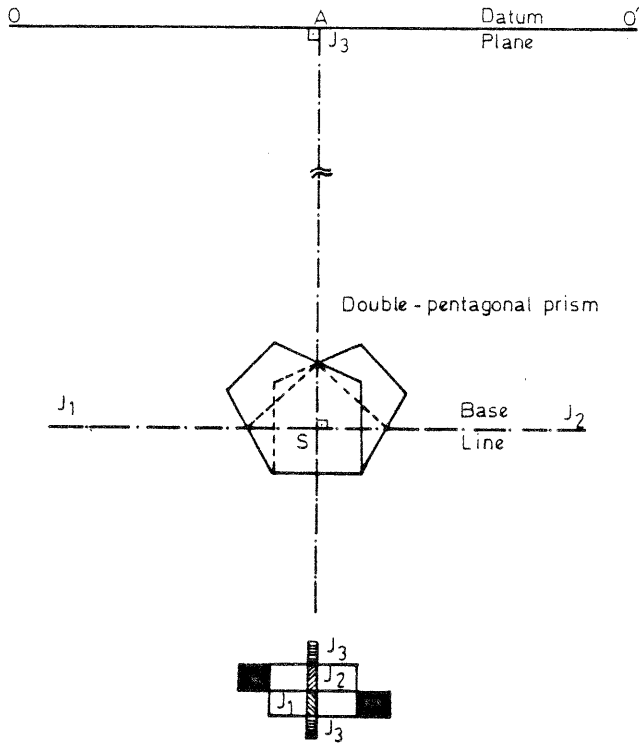


Figure 6. Geometry of the parallel orientation using double-pentagonal prism.

A practical accuracy test was done with the ZEISS SMK 120 (Oberkochen) Stereometric Camera. Three targets were placed on the same plane. A geometrical layout of the test area is shown in Figure 7. There are four meters between the targets and SMK 120 Stereometric Camera was set up ten meters away from the targets plane. Orientation was made with the explained auxiliary apparatus. The pair of photographs were measured in ZEISS PSK2 (Oberkochen) Stereocomparator. The comparator coordinates and transformed image coordinates are summarized on the Table 1.

Object space coordinates of the target points can be calculated with the parallax equations and these are:

$X_A = 4685.399$ mm ; $Y_A = 10030.914$ mm

$X_B = 696.255$ mm ; $Y_B = 10043.388$ mm

$X_C = -3277.939$ mm ; $Y_C = 10065.642$ mm

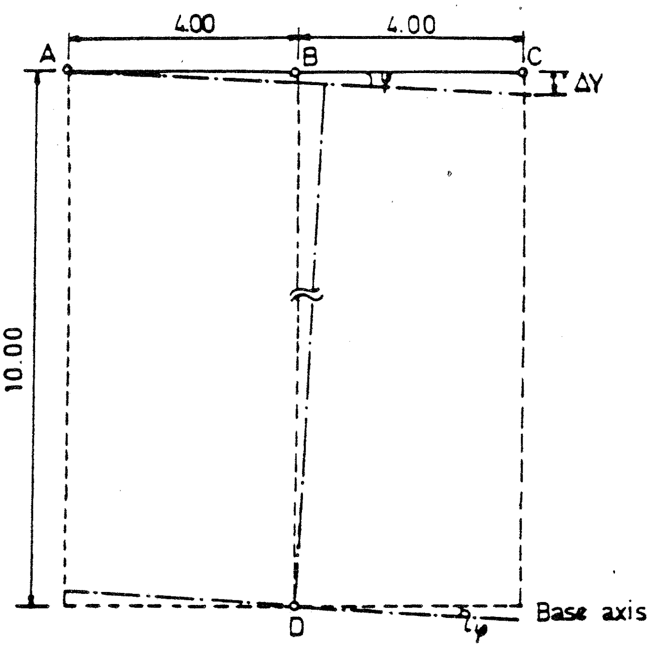


Figure 7. A geometrical layout of the test area.

Rotation of the object from datum plane can be calculated according to the geometrical structure, shown in the Figure 7.

TABLE 1. Comparator and transformed image coordinates

| Point | | Comparator coordinates | | Image coordinates | |
|---|---|---|---|---|---|
| | | x | z | x | z |
| Left Photo | A | 507.244 | 499.548 | -28.292 | -0.424 |
| | B | 531.340 | 500.054 | - 4.199 | 0.197 |
| | C | 555.272 | 499.462 | 19.725 | -0.282 |
| Right Photo | A | 501.364 | 498.716 | -21.046 | -0.505 |
| | B | 525.446 | 499.421 | 3.038 | 0.092 |
| | C | 549.356 | 498.996 | 26.946 | -0.441 |

Consequently, the achievable accuracy can be

$$\tan\phi = \frac{Y_c - Y_A}{X_c - X_A} = \frac{34.728}{7963.338} = 0.00436$$

$$\phi = 0.2776^{g}$$

for the $\phi$ rotation. The expected $\phi$ rotation error can be calculated according to equation (2.1.6) or to equation (2.2.7). For example: if $d\phi$ is calculate with equation (2.1.6), expected $\phi$ rotation error can be

$$d\phi = \frac{0.5\ 60.56\ 63.66}{80\ 50}$$

$$d\phi = 0.40^g$$

for the SMK 120 Stereometric Camera. This value is bigger than the achievable accuracy.

## CONCLUSION

In this paper, new displacement equations of object point due to $\phi$ rotation of the object plane accprding to the datum plane were presented. Focal planes can be made parallel to the datum plane with the help of this proposed auxiliary apparatus. Then, sdale and displacement errors, orthogonal and affinity errors would be minimized on the graphical outputs of analogue plotting with the pair of photographs which are obtained in adequate position of the normal case of stereophotogrammetry.

## REFERENCES

Hallert, B., 1960. Photogrammetry. McGraw-Hill Book Company, INC. New York : 118-119.

Meyer, R., 1970. Aufnahmedispositionen zur stereophotogrammetrischen herstellung von aufrissen. Vermessungstechnik. 18 Jg. Heft 6 : 219-223.

Muftuoglu, O. and Aytac, M., 1980. Calibration of non-metric cameras from a single photograph and pair of photographs. International Archives of Photogrammetry. Volume XXIII, Part B5 : 506-514.