OBJECT SPACE LEAST SQUARES CORRELATION

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ABSTRACT

A method of least squares correlation (LSC) in object (ground) space has been recently developed at Helava Associates Incorporated (HAI). Motivation for this development came from the realization that all information on photographs is inherently in the object space. Transformation of correlation and image interpretation problems to object space leads to a unified approach where all available data is referenced to the same basis. In this paper we discuss the concept of "groundel", defined as the ground (object) space analogue to "pixel" in image space. We then proceed to describe the general principles and formulations of least squares groundel correlation (LSGC) performed in the object space. The ability of LSGC to combine information from several images, groundel by groundel, makes it a potentially powerful tool for detection and elimination of disturbances within the correlation window. This permits reduction of the size of the window, in some cases down to a single groundel. Toward the end of the paper we discuss the application of LSGC to the measurement of point elevations and tie points for aerial triangulation. We expect the groundel concept to eventually become useful as a means of merging automatic correlation and feature extraction into one integrated process.

INTRODUCTION

Ability to correlate images automatically, accurately, and reliably is key to efficient extraction of geometric data from photogrammetric images. Successful image correlation permits derivation of orientation parameters of the images, determination of terrain elevations, and production of orthophotographs and stereomates. Least squares correlation is one of the most interesting and significant concepts brought to bear on correlation problems in recent years. It provides a generalized approach by offering a mathematically tractable way of using multiple input data sets and solving for multiple parameters. In addition, it gives an access to details of the correlation process, because each pixel in the correlation window can be handled individually. This offers unprecedented visibility of operations as well as flexibility in the design of solutions. Furthermore, LSC converts the correlation process to an adjustment problem to which photogrammetrists can apply familiar mathematical and statistical principles.

Image matching by correlation is conventionally performed in the image domain. Principles of image-to-image least squares correlation have been published by several authors [1] to [7]. Least squares correlation in object space is essentially similar to image-to-image correlation. However, it starts from a slightly different point of view, namely that the origin of everything recorded on the images is inherently in the object space and therefore automatic correlation (least squares or not), photo interpretation, and AI-processes should be integrated by performing all these functions in a mutually supportive fashion in the object space. In recent years, Helava Associates Incorporated (HAI) has developed correlation methods performed in object (ground) space.

THE GROUNDEL CONCEPT

As a part of the effort to develop object space correlation techniques, the concept of "groundel" (ground element) has emerged and proven useful. The groundel is a unit in object space similar to the "pixel" in image space. To visualize the groundel concept, imagine the terrain checkered into small squares, each positioned in elevation to represent the average elevation over its area. In addition, each groundel contains information about the reflectance, color, etc. of the piece of ground (object) it represents. (In ambiguous cases two "panels" at different elevations are permissible.) The groundels are approximately the size of image pixels projected to the ground. Each groundel has three-dimensional center coordinates, as well as ascribed attributes, such as slope, radiometric characteristics, and (hypothetical) intrinsic "density". Larger groundels may be built up hierarchically by low pass filtering and aggregation of characteristics. Hierarchical sets with scale ratios 2^n or 2^{2n}; $n = 1,2...5$ have been found useful in correlation over difficult terrain.

The groundel concept is abstract. One can't see groundels delineated on the ground the same way one can see the pixels on the image. Because it's abstract, the groundel system is easily adaptable. It can be oriented in epipolar direction or aligned with the ground coordinates, and the size of the groundels may be selected over a wide range, provided it is not much smaller than the projected pixel.

The groundel concept was originally used in conjunction with conventional correlation methods, specifically as a part of Hierarchical Relaxation Correlation (HRC). Since 1985 it has been combined with least squares correlation (LSC). The results have been very gratifying. This paper discusses some of the basic concept involved. Wrobel [9] has independently developed similar concepts.

GROUNDEL CORRELATION

The basic tenet of groundel correlation is that image densities (radiometric responses of the sensor) corresponding to each groundel can be analytically computed, if all pertinent geometric and radiometric parameters (including groundel reflectance etc.) are known. Errors in the predictions are due to lack of knowledge of exact values of the quantities involved. We wish to use the method of least squares and a set (or sets) of differences between predicted and recorded densities to determine a set (or sets) of unknown quantities or improvements to their approximate values used in the analytical prediction process. The "correlation" aspect of this results from formulation of the least squares problem such that its solution gives information on image conjugacy.

Basic Formulation

Let the sets $\{X,Y,Z\}_i$ and $\{D\}_i$; i= 1,2,...I; represent positions and ascribed intrinsic "densities" of groundels in object space. Multiple imaging produces corresponding sets $\{x,y\}_{i,j}$ and $\{D\}_{i,j}$; i=1,2,...I; j= 1,2,...J; representing J images. The imaging is assumed continuous, broken into an (r,s) matrix of discrete "pixels" by on-line or off-line digitizing. The following transformations apply:

$$(1a) \qquad \{x,y\}_{i,j} = G_j\{X,Y,Z\}_i;$$

$$(1b) \qquad \{r,s\}_{i,j} = S_j\{x,y\}_{i,j}$$

$$(2a) \qquad \{D\}_{i,j} = R_j\,\{D\}_i;$$

$$(2b) \qquad \{D\}_{i,j} = \{@\{D_{r,s}\}\}_{i,j};$$

$$(2c) \qquad \{D\}_i = R_j^{-1}\{D\}_{i,j};$$

$$i = 1,2,...I; \qquad j = 1,2,...J;$$

G_j and R_j denote geometric and radiometric transformations, respectively, $\{x,y\}_{i,j}$ stands for image coordinates of groundel i on image j and $\{r,s\}_{i,j}$ corresponding coordinates in the digitized "pixel" system, $\{D\}_{i,j}$ means "density" (radiometric response) of groundel i on image j, S_j is equivalent to interior orientation (combined with a change of units), $\{D_{r,s}\}$ represents a set of digitized pixels surrounding (x,y), and @ means re-sampling and associated filtering. In general, @ is not reversible. However, re-sampling using the sinc-function regenerates original image densities.

Note that G_j and R_j are groundel independent. Groundel dependencies in R_j are introduced later as Δ-terms.

Both transformations are unambiguous and reversible only when the object surface is "smooth". Smoothness in this context means that the surface is continuous and no occulatations can occur. The smoothness restriction is a tacitly accepted limiting factor in all image space correlation schemes. As will be seen, the groundel correlation concept overcomes this restriction.

In principle, $\{X,Y,Z\}_i$ may be any set of groundels; the set may be regular or irregular, continuous or discontinuous dense or sparse, clustered or dispersed. Each member of the set $\{X,Y,Z\}_i$ has associated members in sets $\{x,y\}_{i,j}$ and $\{D\}_{i,j}$; $i=1,2,...I$; $j= 1,2,...J$. Therefore, formulations based on groundels may be used to describe a wide variety of photogrammetric problems, ranging from global block processing with immense number of groundels to matching of images corresponding to a single groundel.

In practical photogrammetry, G_j a scan be readily determined to a fraction of a groundel. Modelling of R_j requires detailed knowledge of complex physical phenomena with a large number of parameters. Perfect knowledge of all these parameters is virtually impossible. We note, however, that the definition of R_j is tied to the definition of $\{D\}_i$, and that measurement of the parameters of R_j is not important for mapping applications. For ordinary aerial photographs $\{D\}_i$ may be defined such that R_j is approximately a unity transformation for all j. For example,

(3a) $D_i = D_{i,j} = i,$ or

(3b) $D_i = (1/J)\Sigma D_{i,j};\ j = 1,2,...J;$ then

(4) $R_j = 1 + \Delta R_j;$

The correction term ΔR_j is a function of groundel characteristics (e.g. slopes), and variations in radiometric parameters, such as offset, gain, and linearity from image to image and within the images. In terms of densities:

$$(5)\quad D_{i,j} = D_i + \Delta D_{i,j} + \Delta D_j + \Delta_{g,j} D_{i,j}$$

$$i = 1,2,...I; \qquad j = 1,2,...J$$

D_i is defined according to (3a) or (3b), $\Delta D_{i.j}$ represents groundel characteristics, ΔD_j is an offset, and Δg_j stands for gain and linearity corrections. We note that all the Δ-terms in (5) change at slow rates and can be precomputed or calibrated to some level of accuracy. The simple strategy of histogram equalization gives excellent compensation. Known illumination, and observation angles combined with groundel slopes (if known) can be used to refine density compensation further. However, some residual error will remain.

We combine the residual errors to single term Δ_R and get:

(6a) $D_{i,j} = D_i + \Delta_R;$

(6b) $R_j = 1 + \Delta_R/D_i;$

Δ_R is a random variable representing uncompensated density imperfections. It is the fundamental limiting factor in least squares correlation.

Least Squares Formulation

Let us imagine an area of smoothly rolling terrain having a wide spectrum of density variations and covered by several (J) geometrically-perfect, exactly-oriented photographs. In this ideal setting "all rays intersect", giving a set of groundels $\{X,Y,Z\}_i$; D_i;=1,2,...I, and corresponding images $\{x,y\}_{i,j}$; $D_{i,j}$; i=1,2,...I; j=1,2,...J.

Suppose we select one groundel i=k, and change Z_k by a small amount Z_k. Transformations (1a) cause the images of groundel i=k move along their respective nadir lines by small amounts. Transformations (1b) and (2b) cause $D_{k,j}$ to change by $\Delta D_{k,j}$. The ratios $\Delta D_{k,j}/\Delta Z_k$ approach derivatives $\partial D_{k,j}/\partial Z^k$ as ΔZ_k is decreased, even though the image functions are non-analytical. Thus, the perturbation method can be used to linearize non-analytical image functions for setting up least squares equations.

Let us now turn the problem around and assume that Z_k is in error by a small amount ΔZ_k, and we want to measure it using available density information. Due to erroneous Z_k, density discrepancies can be observed at groundel i=k. To explain the discrepancies, we use perturbation to find the dependencies between density changes and changes in Z_k; that is, we form $\{\partial D_{k,j}/\partial Z_k\}$; j=1,2,...J. We can then write simultaneous equations:

(7) $(\partial D_{k,j}/\partial Z_k)\Delta Z_k = D_k - D_{k,j} + \Delta_R;$ j=1,2,...J

Solving for ΔZ_k (simple, since there is only one unknown) gives the required answer. The process may by considered as a multi-image "correlation" of groundel k.x

In the example above, ΔZ_k is groundel specific; it belongs to groundel i=k only. However, the same reasoning applies when many, or even all, groundels are in error by the same amount ΔZ. Equations (7) then become:

(8) $(\partial D_{i,j}/\partial Z_i)\Delta Z = D_i - D_{i,j} + \Delta_R;$ i=1,2,...I;
j=1,2,...J

Let a cluster of groundels (K) be defined by a subset i=k,k+1,...K. The solution of (8) may in that case be considered "correlation" over a "window" K.

Returning to the idealized situation with all Z values correct, we denote the orientation elements of the photographs $\Omega_{q,j}$; q=1,2....Q; j=1,2,...J. We now change the orientation element q=m of photograph j=p, $\Omega_{m,p}$, by a small amount $\Delta\Omega_{m,p}$. That affects the densities of all groundels $D_{i,p}$; i=1,2,...I; j=p. We can measure $\Delta\Omega_{m,p}$ by perturbing $\Omega_{m,p}$ to establish the linear dependencies, forming equations:

$$(9) \qquad (\partial D_{i,p}/\partial\Omega_{m,p})\Delta\Omega_{m,p} = D_i - D_{i,p} + \Delta_R;\ i=1,2,...I;\ j=p$$

and solving for $\Delta\Omega_{m,p}$. Again, there is only one unknown in (9). By specifying m=1.2....Q, (9) expands to Q unknowns and covers the complete orientation of photograph j=p.

To summarize, we write a combination equations:

$$(10) \qquad (\partial D_{i,j}/\partial Z_i)\Delta Z_i + (\partial D_{i,j}/\partial\Omega_{q,j})\Delta\Omega_{q,j} \ D_i - D_{i,j} + \Delta_R$$

$$i=1,2,...I;\quad j=1,2,...J;\quad q=1,2,...Q$$

Equations (10) include corrections to all groundel elevations and orientation elements of all photographs. It is a form of "global least squares correlation", suitable for trimming an already existing, near perfect photogrammetric solution.

The process described in (10) fails if J<3. There are more unknowns than observations. The dilemma can be solved by introducing functional relationships between Z-values; for example in the form of continuity constraints [7], facets [9] , or a higher order surface [8] (see also (11) below).

Before (10), or any of its subsets, can be used, a near perfect photogrammetric solution must be somehow produced. For that, one can break the problem into triangulation, orientation, point wise correlation, etc. as is done conventionally. This can be done readily, with high geometric accuracy and efficiency, and with quite good elevation measurement by correlation. Alternatively, one can apply the global solution hierarchically. The latter approach means starting the process with very large groundels and correspondingly filtered and minified photographs. The concept has been used in measurement of terrain elevation matrices, with good result. It should work in the global solution, but we have not tested it yet.

The global solution is interesting, but in our judgement of little practical value. Besides being computationally overly demanding, it has a significant technical flaw: It does not address the main problem in photogrammetric correlation; that is, resolution of ambiguities and occultations.

Conventional techniques can produce essentially perfectly oriented models by measurements made at a relatively small number of points, and perform excellent correlations when ambiguities and occultations are absent. We have presented the global formulation above out of general interest, and to provide background for discussions focused on certain subtasks where the least squares groundel correlation can render valuable help.

MEASUREMENT OF TERRAIN ELEVATIONS

Measurement of terrain elevations on well-oriented stereo models is economically the most important task for automatic correlation. Advanced methods of conventional correlation, such as the Hierarchical Relaxation Correlation, are capable of developing good terrain models quite efficiently. However, these techniques can't be used in detail analysis of the object surface, because they do not provide any visibility into conditions inside the correlation window. Least squares groundel correlation (and LSC in general) offer such visibility, thus giving means for refinement of results of conventional correlation. Obviously, hierarchical groundel correlation can also develop the initial terrain models; however, in that capacity it does not seem to offer any fundamental advantage over HRC.

Equation (7) indicates that measurement of elevations of individual groundels is possible under the right conditions. The primary conditions are existence of good initial estimates, valid derivatives, and sufficiently large J. Obviously, J=1 fails, but J=2 can work, at least theoretically. The process becomes increasingly robust with increasing J. J=4, obtained by using 60% side overlap, may be robust enough to be of practical value. However, we have no experimental verification of this hypothesis at this writing.

Robustness of the measurement can be improved also by increasing the number of groundels included in the measurement. To do that, the elevations of the groundels must be tied together by some functional relationship. In equation (8) the functional relationship is one of equality; all groundels within the window have the same elevation correction. This simple paradigm is widely applicable under the smooth surface assumption. We wish, however, to develop a more universal paradigm. Let $\{X,Y,Z\}_i$; i= 1,2,...I; I>>1; be a set of groundels over an area A. Our objective is to find Z_o, with the groundel i=0 essentially at the center of A. To accomplish that we postulate a relationship between the elevations of the groundels:

$$(11) \quad Z_i = f^n(X_i-X_o,Y_i-Y_o); \quad n = 0,1,2,...N, \; i=1,2,...I$$

In (11) f^n is an Nth order t-dimensional function with a set of coefficients $\{a\}^n$ describing the object surface. The zero order term indicates how much Z_O needs to be changed. The coeficient should be selected such that the rms value of density discrepancies after application of f^n is Δ_R. Too small an rms means that f^n has absorbed radiometric errors, while too large an rms indicates that the order on f^n is too low. However, this is an idealized situation. Realistically, the presence of some amount of imperfection in object modelling must be recognized. Then, expanding on (6a) we get:

$$(12) \qquad D_{i,j} = D_i+\Delta_R+\Delta_T; \; i=1,2,...I; \; j=1,2,...J$$

where Δ_T represents terrain modelling error. We now modify (8) to get:

$$(13) \qquad [(\partial Z_i/\partial\{a\}^n)\,(\partial D_{i,j}/\partial Z_i)]\Delta\{a\}^n = D_i-D_{i,j}+\Delta_R+\Delta_T$$

$$i = 1,2,...I; \quad j = 1,2,...J; \quad n = 0,1,2,...N$$

In (13) $\{a\}^n$ denotes the unknown coefficients of f^n and $\Delta\{a\}^n$ the corrections to them. Because the density functions are non-analytical, the $\partial D/\partial Z$ derivatives must be determined by perturbation. The $\partial Z_i/\partial\{a\}^n$ derivatives are analytical.

The terrain modelling approach expressed in (13) could be extended ad infinitum by increasing N. However, the terrain surface is not analytical and the use of large N, or any other stratagem that relies on continuity, is futile. The problem is that the major error source in automatic correlation (LSC included) is the presence of unpredictable and highly non-analytical "terrain noise"; that is, bushes, trees, boulders, crevices, houses, hedges, telephone poles, and other such natural and man-made objects. The main objectives, and most valuable contributions, of groundel correlation are to detect such disturbances and eliminate their effects.

In terms of conventional correlation, the presence of disturbances manifests itself as incoherence of correlation data. The conventional correlation techniques detect the incoherence as a low correlation coefficient, but can't indicate where the problem is. Therefore, the window size must be increased until a sufficient level of coherence is achieved. In least squares groundel correlation, elevation residuals can be derived for all groundels and analyzed for consistency. Detected inconsistencies can then be dealt with in a variety of ways. For example, the window size can be adjusted down until the discrepancies can be considered arising from radiometric noise. Alternatively, the groundels with large residuals could be simply removed from the solution to get a result that amounts to a majority vote.

The most sophisticated solution is to develop a geometric interpretation of the residuals, with proper accounting for perspective displacements, occultations, and shadows. This approach offers great potential for measurement of terrain microphotography and resolution of ambiguities. We hope to report soon on experiments with this approach.

MEASUREMENT OF TIE POINTS FOR AERIAL TRIANGULATION

One of the most important tasks in aerial triangulation is the transfer of tie points from photograph to photograph. This is customarily done pairwise, but in principle it is a multi-image correlation task. In this application, J is always at least three, with J=5 common, and J=9 possible when 60% side overlap is used. Tie point correlation is an ideal application of least squares correlation concepts.

However, the application of these concepts does not per se guarantee excellent results. We are reminded of the ever possible presence of disturbances, and their effects on the measurements. Fortunately, the least squares approach is capable of detecting the disturbances and reducing their effects. In triangulation, the reduction should be by selection. Since the tie point need not be exactly at any given location, the logical way to use the disturbance detection capability of least squares is to select a favorable tie point location. The selection can be carried even further.

The least squares technique can measure also the flatness, levelness, and signal content of candidate tie point areas. An ideal area has no disturbances, is horizontal, and has high signal content. A horizontal piece of terrain has minimum shape distortion on multiple images.

It may seem off hand that groundel correlation is unsuitable for measurement of tie points, since no oriented model is (normally) available. However, there are ways around that problem. One way is to perform a pre-triangulation and use the least squares groundel correlation to do point refinement, including selection of the final point. Formulation (13) can be used in the selection and measurement processes. Only zero and first order terms should be used. Areas with disturbances can be rejected and the point selected where high density slopes and low terrain slopes coincide. (1a) is applied to get the image coordinates for triangulation. If desirable, a "mini" global solution (see (10)) can be carried out with small clusters of groundels around each tie point.

Another way to use groundel correlation in tie point measurement is to establish small individual groundel systems around each tie point. The groundel systems should be oriented approximately parallel to the flight lines. Since the photographs are not oriented, G_j and S_j transformations are not known, and matching image areas may be difficult to find.

To overcome this, we select one of the images as the master image and use HRC to achieve initial matching. The HCR algorithm includes first order shaping, permitting two-way mapping between (r,s) systems of master and slaves images. In effect, the slave images become rectified to the master image. One can then use guesses of coordinates of the air stations, f/H, and unity rotation matrices to form approximate G_j; j= 1,2,...J. Over the small tie point area the effects of orientation elements are shifts which can be absorbed into the guesses of air station coordinates. To continue, formulation (13) could be used, as described above.

CONCLUDING REMARKS

We have described and discussed a variation of least squares correlation we call least squares groundel correlation, or groundel correlation for short. It differs from other variations in that it is performed in object space by matching densities assigned to "groundels". We have emphasized the use of groundel correlation as a "trimming" process applied on results obtained by using conventional methods. The rationale is that conventional techniques are quite efficient, but need help for final refinements of their results. In this, the ability of groundel correlation to process each groundel individually, accommodate multiple images, and employ mathematically rigorous processing offers means for significant sharpening of automatic measurements. We expect that eventually the groundel concept will be found advantageous also in automatic feature extraction, thus helping to merge geometric and interpretative analyses into one unified process.

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