Multi Plate, Multi Exposure and Multi Free Exterior Orientation Stellar Calibration for Metric Camera

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ABSTRACT

By studying stellar calibrtions of metric camera, the author derived the conclusion that the initial values of exterior orientation α_o、δ_o may be selected in a large field , and gave the method how to determine the initial values of other parameters . Thus formed a new simple way of stellar calibration — Multi Plate, Multi Exposure and Multi FREE Exterior Orientation Stellar Calibration , which has, compared with traditional methods, the following advantages:

1. The camera exterior orientations need not to be measured, while its optical axis may be set in any directions within the vetical angle greater than 20° .
2. Any movement of the camera when exposuring has no effect on the calibration results.
3. The important need of precise clocks in traditional methods will be unnecessary in this new method.
4. Should not consider the effect of atmospheric refraction, precession, nutation and sidereal proper motion.
5. The computation for adjustment can be carried on separately, thus can be carried out on a micro-computer.

An experiment was made on with metric camera UMK 30—1318. for compatation, POWELL interation method was used on computer IBM—PC will language FORTRAN—77. The results of the experiment verified that the new calibration method is effective.

I . Background of Stellar Calibaration

The approach of the calibration usually inclucles

1. Taking pictures of some fixed stars with the camera so as to get the plate coordinates of these pitures.

2. Composing the relation equations between the plate coodinates and the star coodinates.

3. Solving these equations (also including the parameters of camera) in order to obtain those camera parameters being calibrated.

See figure 1, we suppose first that 0— x,y,z is the plate coordinate system (PCS) , and 0—X ,Y ,Z the celestial sphere coordinate system(CSCS) , where 0 is the optical center of the camera lens and 0 the fiducial center of picture. 0o the optical axis of the camera lens, α_o and δ_o the right ascension and declination of the optical axis respectively.

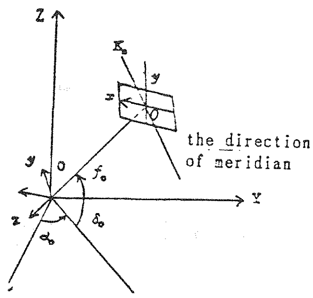

Figure 1

Then, let α_i be the right ascension, δ_i the declination of a star in CSCS when exposuring, and x_i, y_i its picture coordinates with respect to PCS. the z-axis of PCS is in coindance with $f_\circ$, the fixed focal length of the camera.

Therefore, from [1], we have

$$x_0 - f_0 \frac{a_1 A_i + b_1 B_i + c_1 C_i}{a_3 A_i + b_3 B_i + c_3 C_i} + \triangle x_i - x_i = 0 \tag{1}$$

$$y_0 - f_0 \frac{a_2 A_i + b_2 B_i + c_2 C_i}{a_3 A_i + b_3 B_i + c_3 C_i} + \triangle y_i - y_i = 0$$

where $x_\circ$ $y_\circ$ $-f_\circ$ are so-called the interior orientation parameters of the camera, a_1 b_1 c_1 a_2 b_2 c_2 a_3 b_3 and c_3 the elements of the transformation matrix from [α_i δ_i] to [x_i y_i $-f_\circ$], and these elements being the functions of $\alpha_\circ$ $\delta_\circ$ $k_\circ$, A_i, B_i, C_i are the functions of α_i and δ_i, while $A_i = \cos\alpha_i \cos\delta_i$, $B_i = \sin\alpha_i \cos\delta_i$, $C_i = \sin\delta_i$, and $\triangle x_i$ and $\triangle y_i$ are the effections due to lens deformations, they are expressed in some paper [2] by

$$\triangle x = \overline{x}\,(k_1 r^2 + k_2 r^4 + \cdots) + [p_1(r^2 + 2\overline{x}^2) + 2p_2 xy] + (1 + p_3 r^2)$$

$$\triangle y = \overline{y}\,(k_1 r^2 + k_2 r^4 + \cdots) + [p_2(r^2 + 2\overline{y}^2) + 2p_1 xy] + (1 + p_3 r^2)$$

where

$\overline{x} = x_i - x_s$, $\overline{y} = y_i - y_s$, $r^2 = \overline{x}^2 + \overline{y}^2$ and k_1 k_2 $k_3 \cdots\cdots$ p_1 p_2 $p_3 \cdots\cdots$ are the deformation parameters.

when measuring images of stars, we eventually obtain x_i, y_i with errors, say x_i', y_i', if

$x_i' + V_{xi} = x_i$, $y_i' + V_{yi} = y_i$, equation (1) will become

$$V_{x_i} = x_0 + \triangle x_i - f_0 \frac{a_1 A_i + b_1 B_i + c_1 C_i}{a_3 A_i + b_3 B_i + c_3 C_i} - x_i'$$

$$V_{y_i} = y_0 + \triangle y_i - f_0 \frac{a_2 A_i + b_2 B_i + c_2 C_i}{a_3 A_i + b_3 B_i + c_3 C_i} - y_i' \tag{2}$$

for one image of a star, we can list such an equation pairs, so, for n images of the stars, we can then, establish n eguation pairs as below.

$$V_{x_1} = x_0 + \triangle x_1 - f_0 \frac{a_1 A_1 + b_1 B_1 + c_1 C_1}{a_3 A_1 + b_3 B_1 + c_3 C_1} - x_1'$$

$$V_{y_1}=y_0+\Delta y_1-f_0\,\frac{a_2A_1+b_2B_1+c_2C_1}{a_3A_1+b_3B_1+c_3C_1}-y_1'$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$V_{x_n}=x_0+\Delta x_n-f_0\,\frac{a_1A_n+b_1B_n+c_1C_n}{a_3A_n+b_3B_n+c_3C_n}-x_n' \qquad (3)$$

$$V_{y_n}=y_0+\Delta y_n-f_0\,\frac{a_2A_n+b_2B_n+c_2C_n}{a_3A_n+b_3B_n+c_3C_n}-y_n'$$

In order to obtain the camera parameters(x_0, y_0, f_0, x_s, y_s, k_1, k_2, k_3 ······), the equations above are usually solved according to Least Square Criterion :

$$\sum_{i=1}^{n}(Vx_i^2+Vy_i^2)=\text{minimum}.$$

For determining these parameters, we should know the exterior orientation elements (EOE) α_0 δ_0 k_0, the coordinates of star α_i, δ_i and the picture coordinates of the star images.

In previous calibrations [1], [3], either the EOE must be measured or the optical axis must be set to the direction toward the zenith in order to determine α_0 δ_0 k_0, Besides, precise clocks must be used for recording the universal time(UT) of exposuring and also the latitude ϕ and longititude λ of the exposuring station must be given. with α_0, δ_0, k_0, UT, λ and ϕ, the correct coordinates of star α_i, δ_i when expossuring can be known.

II. FEOE stellar calibration method.

As a matter of fact, our calibration is facing such a problem, that is , to map the camera parameters from a structure of a star group. so, if we know the relative positions of the stars, we can determine the parameters including α_0 δ_0 k_0, here, we are taking α_0, δ_0, k_0 as unkown variables.

It is easy to determine the relative positions of stars during exposuring, we may directly use the values of α_i δ_i in the date near that of exposuring of the stars looked up in the astronomical almanac, since the right ascension α_i and delination δ_i of stars vary in relative position so slightly that in several days its changes are less than 1", for the positions' absolut changes, we can take them as to be the varieties of α_0 and δ_0.

The values of α_0, δ_0 are determined by means of the theory as described below (see part III), the camera parameters can be solved with α_0 and δ_0 simultaneously.

III. Convergence of α_0 δ_0

It is affirmative that α_0 and δ_0 can be solved from the equations composed by the coordinates of the stars, since, after gived the initial values of the parameters except α_0 and δ_0, Vxi and Vyi in equations(3) and hereafter

$$f(\vec{x})=\sum_{i=1}^{n}(V_{xi}^2+V_{yi}^2)$$

is only a function with two varibles, which can be expressed by

$$f(\vec{x}) = f(\alpha_0, \delta_0)$$

with mathematic method, we can prove that

1. there exist the values of $\Delta\alpha_o$ and $\Delta\delta_o$ which make

$$f(\alpha_0+\Delta\alpha_0, \delta_0) > f(\alpha_0, \delta_0)$$

$$f(\alpha_0, \delta_0+\Delta\delta_0) > f(\alpha_0, \delta_0)$$

$$\text{and} \quad f(\alpha_0+\Delta\alpha_0, \delta_0+\Delta\delta_0) > f(\alpha_0, \delta_0)$$

2. the larger the values of $|\Delta\alpha|$ and $|\Delta\delta|$, the lager the value of $|\Delta f(x)|$, i.e

if $|\Delta\alpha''| > |\Delta\alpha'| > |\Delta\alpha_o|$

then $f(\alpha_o+\Delta\alpha'', \delta_o) > f(\alpha_o+\Delta\alpha', \delta_o)$

and if $|\Delta\delta''| > |\Delta\delta'| > |\Delta\delta_o|$

then $f(\alpha_o, \delta_o+\Delta\delta'') > f(\alpha_o, \delta_o+\Delta\delta')$

3. the change of $f(\vec{x})$ due to the change of $\Delta\alpha$ is independent of the value of δ, and the chang of $f(\vec{x})$ due to that of $\Delta\delta$ is independent of the value of α.

4. if $|\alpha 2| > |\alpha 1| > |\alpha_o+\Delta\alpha_o|$

then $f[\pm(|\alpha 2|+|\Delta\alpha|), \delta] > f[\pm|\alpha 1|+|\Delta\alpha|, \delta]$

and if $|\delta 2| > |\delta 1| > |\delta_o+\Delta\delta_o|$

then $f[\alpha, \pm(|\delta 2|+|\Delta\delta|)] > f[\alpha, \pm(|\delta 1|+|\Delta\delta|)]$

From above, we can deduce that when we search α_o and δ_o from a random initial value of α_o δ_o, say $\alpha oo'$, $\delta oo'$, we may first search along the direction $|\Delta\alpha|$ increase or decrease, and the last value must be α_o' ($\alpha_o' = \alpha_o+\Delta\alpha_o$), and it is the same for δ_o. This method of search is just the powell interation method.

Ⅳ. An experiment with UMK 30-1318 close range photogrammetric camera by FEOE stellar calibration method.

1. A brief introdution

with the same camera which has a solid focal length, in three different directions, at different UT, we took several photos of a star field which includs Lyra and Aquila. there were three exposured plates 6 stars images on plate NO.1, 3 on NO.2 and 4 on NO. 3. The star images exposured at every 6 different UT on every plate were measured.

The picture coordinates of the images of stars exposured at the same UT were treated as a basic data group (BG), for every basic group is related to a group of exterior orientation elements α_o δ_o k_o (j=1-6), and three BG from the three plates composed an adjutment group(AG), so we got altogather 6 adjustment groups.

composing the equations like (2) with each AG and computing with powell iteration method, we get the parameters :

$$\alpha^j_{o1} \quad \delta^j_{o1} \quad k^j_{o1} \quad \alpha^j_{o2} \quad \delta^j_{o2} \quad k^j_{o2} \quad \alpha^j_{o3} \quad \delta^j_{o3} \quad k^j_{o3}$$

$$x^j_0 \quad y^j_0 \quad f^j_0 \quad x^j_s \quad y^j_s \quad k^j_1 \quad k^j_2 \quad k^j_3 \cdots\cdots$$

where the initial values of α_o, δ_o and k_o were determined according to the method described in Ⅲ, by taking $x_o=0$, $y_o=0$, $f_o=300$mm and $k_o=0$ the results of each AG are listed in table 1.

2. The computations of the parameters and the estimations of their accuracies.

Based on table 1, we can estimate the errors of mean squares with unit weight, say it being m_{oj} (j=1-6)

$$m_{oj} = \pm \sqrt{\frac{\sum_{i=1}^{n} (V_{x_i}^2 + V_{y_i}^2)}{n_j - t_j}}$$

By calculating the weighted average of the parameters from the six AG , we have

$$x_o = \frac{\sum \frac{x_{oj}}{m_{oj}^2}}{\sum \frac{1}{m_{oj}^2}} = \frac{3.4004}{0.15188} = 22.3\mu$$

$$y_o = \frac{\sum \frac{y_{oj}}{m_{oj}^2}}{\sum \frac{1}{m_{oj}^2}} = \frac{-5.7107}{0.15188} = -37.6\mu$$

$$f_o = \frac{\sum \frac{f_{oj}}{m_{oj}^2}}{\sum \frac{1}{m_{oj}^2}} = \frac{46.0738}{0.15188} = 303.356\text{mm}$$

It is true that these values differ very much from the tabled values X_o (10μ), y_o (-10μ), f_o (303.35mm), but the effictivenss of FEOE stellar method is verified yet, for the differnces between the calibrated values and the tabled values are due to large measuring errors of plate coodinates in practise.

In (3) , the author gives the formula to estimate the parameters' EMS(errors of mean square)with unit weight,which is:

$$m_o^2 = \frac{\Sigma r_i}{r} + \frac{1}{r} \Sigma \frac{(X_i - X)^2}{m_{oi}^2}$$

For our calibration , it will be

$$m_o^2 = \frac{74}{15} + \frac{1}{15} \times 22666.97 = 156.06\mu^2$$

Therefore, the EMS of each parameter is

$$m_n = m_o \sqrt{\frac{1}{P_x}} = m_o \sqrt{\frac{1}{\frac{1}{\Sigma m_{oi}^2}}} = \pm 32.05\mu$$

V. Conclusions

1. Through mathematical analysis, we found that the exterior orientation parameters α_o δ_o can be convergent in the iteration process, and their iterative initial values may be selected from

$$\alpha_{o o}' \subset (\alpha_o - 90° + \varepsilon, \alpha_o + 90° - \varepsilon)$$
$$\delta_{o o}' \subset (\delta_o - 90° + \varepsilon, \delta_o + 90° - \varepsilon)$$

where ε is the maximum angle of image field of camera.

2. Since α_o δ_o can be convergent, when use FEOE stellar calib method, it needs not to measure the exterior orientation elements α_o, δ_o while the optical axis of camera may be set in any directions. except its zenith distance greater than 70° .

3. The important need of precise clocks in traditional methods will be unnecessary in this new method.

4. Should not consider the effect of atmosperic refraction, (when step 2. is made) precession, nutation and sidereal proper motion.

5. The computation of adjustment can be carried on separately thus can be carried out on a micro-computer.

Table 1 The results of calibration

No. of AG	$x_0(\mu)$	$y_0(\mu)$	f_0(mm)	ΣV^2	n_i	t_i	r_i	$m_{oi}(\mu)$
1	54.75	-13.15	303.313	5094.85	26	14	14	±19.07
2	33.30	-162.29	303.319	1332.60	26	12	14	±9.76
3	176.82	53.75	303.54	1552.10	26	12	14	±10.53
4	220.96	-80.73	303.339	788.54	26	12	14	±7.55
5	-69.90	61.37	303.368	275.34	18	9	9	±5.53
6	-3.60	31.34	303.361	113.53	18	9	9	±3.55

Notes : n_i — the number of variables in each AG.
t_i — the number of necessary variables in each AG.
r_i — $r_i = n_i - t_i$.

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