ALGORITHMS FOR A STRUCTURAL MODEL OF BIPED LOCOMOTION IN THE CONFIGURATION SPACE

Javier Finat
Faculty of Sciences, University of Valladolid
jfinat@nava.tel.uva.es

Working Group MoBiVA Group, Neurotecno Institute

KEY WORDS: Model-based processing, Navigation.

ABSTRACT

Mechanical models for locomotion are strongly hierarchised depending on the gait phase, tasks to be performed and constraints. There is an alternance between different constraints involving to different phases of any locomotion. This alternance suggests a hybrid approach for the algorithms design, with coarse-to-fine models for the automatic selection of rules relative to geometric motion planning, adaptive control and monitoring forces. One introduces doubly connected lists for the geometric data representation and updating of incidence elements between lines supporting actions with the corresponding pointers to points and planes. This geometric model supports the existence of spatio-temporal symmetries linked to spatial rigid transformations into the ambient space for representation and in the architecture for motion simulation, and to temporal alternant patterns for periodic motions. Both of them can be represented in terms of reflections which provide a modular approach to the design and implementation of algorithms. Velocity fields involving to mobile data provide tools for spatial replication and periodicity phenomena and give modular, adaptable and reconfigurable patterns for the algorithms implementation.

1 INTRODUCTION

Locomotion tasks for biped or multilegged robots involve to the generation, transmission and coordination of movements for a regular gait. The design of algorithms for planning and execution of terrestrial locomotion requires to select objectives and to formulate *simplifying assumptions* for the models. The *goal* is to perform smooth periodic patterns including stance and swing phases separated by land-off and ground impact as phase transitions for each leg.

To illustrate these general principles, the concatenation of all these phases is realised in terms of flexion/extensions operations allowing shape changes in articulated mechanisms. So, we forget the dissipation effects and put emphasis on the alternance and periodic movements. So, along a regular walking whereas the left leg is falling in its extension phase under the effect of the gravity force (navigation phase), the right leg is contracting (in the flexion phase) along the stance phase (dynamic reflection). Regular phases correspond to stance and swing with an alternance between active and passive interaction with a wrenches exchange involving to the activation-inhibition of efforts generation and inertial effects holding onto the flexion and extension artificial modules (in correspondence with agonistic-antagonistic behavior in the biological case). Exchanges between kinematics and dynamics hold onto phase transitions (land-off and impact against the ground) which are assumed as instantaneous and modelled as singularities of the model. Along the regular phases, one applies superposition principles for kinematics and dynamics, by adopting a representation in terms of 3D lines.

The diversity of phenomena appearing in a changing dynamics suggests a hybrid approach for the design and implementation of algorithms. The hybrid character is relative to perception-action cycle; another said, it combines a propioceptive information based on geometric models for the behavior analysis and a mechanical model based on motion's equations. To the behavior analysis one can add a structural mechanical model to make easier the tracking, prediction and generation of motions, even in presence of partial occlusions. Furthermore the visual information it is convenient to have models for gait generation in legged mechanisms which provide a mechanical feedback for a more efficient control.

It is important the reusability of routines in software libraries to allow an information transference between different perception-action modules in distributed systems. The hierarchies of this artificial model are symbolically represented in terms of like-tree graphs (including cycles for parallelizable questions), to make easier the maintainance of a queue of internal events associated to locomotion tasks. The information updating must be linked with the control system in an interactive way by some kind of velocity fields. Neural fields are given by potential fields which are active or passive, depending on the motion phase; they are in charge of generating velocities and forces acting onto rotational joints which are translated into twists and wrenches in $\mathcal{W}$. They follow biologically inspired rules for binary activation/inhibition patterns to perform the planned tasks which are translated into switching and tuning behaviors in control processes.

These fields can be used for any other parallel mechanism, s.t. a multifingered hand where one uses a symbolic representation based on cubes with binary vertices to identify typical postures of an artificial hand (Gonzalo-Tasis and D.Sanchez, 2000) ; this representation can support also kinematic and dynamic information as paths onto unions of cubes with variable weights playing the role of local varying curvatures.

Boissonnat et al (1995) develop algorithms for motion planning based on accessible and stable configurations of legged robots (Boissonnat and S.Lazard, 1995). Nevertheless its local efficiency, to update this information in the working space it is necessary to have to our disposal a cellular decomposition of a highly structured scene, and transfer the information relative to possible swings of each leg. Even in this case, the update of this information is too casuistic and some expensive from the computational viewpoint, due to the need of verifying a complex system of constraints. Furthermore, it is not clear how to evaluate different kinds of unstability in legged mechanisms by using only information about the geometry of configurations and working space. The source of unstability in locomotion tasks arises from kinematics and dynamic of multilegged robots, independently of the scene. One needs to design a hierarchised system to diminish the number of verifications, and the introduction of local symmetries to make easier the propagation of elementary patterns along the truss.

There exist another low-level approaches based on ANN (Artificial Neural Networks) or the Fuzzy-Logic (FL) Algorithms, based on logic rules which are very useful for unstructured environments or in absence of a better knowledge of the mechanics of multilegged robots. However, after the training process, the ANN is not reusable when the conditions are changing. The FL-approach does not allow to incorporate mechanical properties of locomotion phases to control, and it requires strong compensating mechanisms due to its ignorance about the mechanical model. Finally, it is difficult to find convergence criteria for optimization processes developed from genetic algorithms.

Thus, in this note, I shall paid attention only to structural models. My approach is related to the adjustment postural strategy (Gorce, 1998). It allows to integrate local and global aspects of the mechanical model, and modify it in terms of vector fields which update mobile information. I shall put more emphasis onto some hierarchies of mechanical aspects which allows us an integrated treatment of kinematic and dynamical constraints in articular coordinates to assert a marginal and dynamic behavior around stable trajectories in locomotion.

To solve the geometric representation problem of multibody systems in working space $\mathcal{W}$, one can introduce adapted coordinated frames associated to mobile references following a hierarchised model associated to multi-points or alternately to configurations of lines which can be extended to motion planning and control. Anyway, the articular and (multi)vector representation are related through the representation of elementary reflections and this provides an easy description of transmission and propagation phenomena along the mechanical architecture going from the configuration space $\mathcal{C}$ to the working space $\mathcal{W}$ (and inversely). The spatio-temporal matching of these mobile references is performed in terms of vector fields, by providing in the way the necessary feedback for the force-position traditional control.

2 THE MECHANICAL MODEL

The general ingredients for the mechanical model are related with the support (configurations and working spaces), the tasks to be performed, variable constraints acting along different phases and devices to simplify the analysis (invariants to identify, cost-benefit functions for optimization processes, etc).

Any multibody system is a collection of kinematic chains linked to a mobile platform. In our case, each kinematic chain is given by a leg with a planar motion, after selecting the value of the corresponding Euler angle at spherical joint at the hip. Each leg is modeled as the coupling of three pendula with two rotational joints (for the knee and heel), and a third spherical joint in the hip for the configurations space $\mathcal{C}$. Onto the working space $\mathcal{W}$ one can introduce mixed optimization criteria given by a homotopy between a general quadratic functional plus a smoothing operator.

Nevertheless the generality of this approach, their meaning is different depending on the context. So, the quadratic functional corresponds in the visual case to relative distance functionals (differences between desired and current position-orientation in model-based vision or disparity in Stereo Vision, e.g.). In the mechanical case it corresponds to the ordinary total energy function if we take control nodes or to the weighted squared norm of the corresponding 6-dimensional twist (angular momenta and linear momenta) if we take the geometry of lines as the framework to develop the kinematics; difficulties appearing with odometry and calibration suggest to use lines instead of nodes for relative positioning. In the same way one can introduce similar quadratic functionals for screws in Geometry and for wrenches in Dynamics. So, the conclusions can be extended for any model based on the geometry of lines (the right framework is the Clifford algebra, but I have no space for this more general approach, here). The analysis for smoothing operator is similar.

2.1 A hierarchised support for mechanics

If we add tasks and constraints to the well-known transference between small movements at joints and motions in ambient working space (Finat and S.Urbaneja, 1999), one can represent it by means of a general commutative Extended Main Analytic Diagram (EMAD in the successive):

$$\begin{array}{ccccccc} J^2\mathbf{R}^n & \rightarrow & J^2\mathcal{C} & \rightarrow & J^2\mathcal{W} & \rightarrow & J^2\mathbf{R}^p \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ J^1\mathbf{R}^n & \rightarrow & J^1\mathcal{C} & \rightarrow & J^1\mathcal{W} & \rightarrow & J^1\mathbf{R}^p \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ J^0\mathbf{R}^n & \rightarrow & J^0\mathcal{C} & \rightarrow & J^0\mathcal{W} & \rightarrow & J^0\mathbf{R}^p \end{array}$$

where each arrow corresponds to geometric, kinematic and dynamic aspects of the mechanics in terms of the configuration $\mathcal{C}$ and working $\mathcal{W}$ spaces related through the natural projection $\pi : \mathcal{C} \rightarrow \mathcal{W}$ and their kth order prolongations given by the k-jets $j^k\pi : J^k\mathcal{C} \rightarrow J^k\mathcal{W}$. So, the horizontal lines concern to relations between forward and inverse geometry, the kinematics (for the first prolongation $j^1\pi$ is represented by the usual jacobian matrix), and the dynamics (for $j^2\pi$ giving the Euler-Lagrange formulation).

Onto this diagram, one represents the *tasks* as paths $\gamma : \mathbf{R^n} \rightarrow \mathcal{C}$ which can be composed with π to give tasks on working space or lifted to successive prolongations, obviously giving the left part of the EMAD. Similarly, the *constraints* can be represented as maps $j^kg : J^k\mathcal{W} \rightarrow J^k\mathbf{R^p}$ depending if they involve to the geometry, the kinematics or the dynamics, giving the right part of the EMAD. Their lifting to $J^k\mathcal{C}$ is performed thanks to the local triviality conditions for the topological fibrations $j^k\pi$.

Furthermore these horizontal relations, one has the canonical projections $p^{k,k-1}_{\mathcal{X}} J^k\mathcal{X} \rightarrow J^{k-1}\mathcal{X}$, which forget the generalized coordinates corresponding to the k-th order derivatives, where $\mathcal{X}$ represents $\mathcal{C}$ or $\mathcal{W}$. So, the vertical arrows concern to the transference between geometric, kinematic and dynamic principles for a parallel robot. This transference is performed in terms of vector fields which (due to the singularities) can present singularities at the adherences of strata. To simplify the spatio-temporal matching, I shall consider only vector fields which can be replicated at such boundaries by means the application of reflections (this can be justified in terms of an equivariant stratification).

This general hierarchised model has its corresponding version for stability and control chart for locomotion, which are not considered here due to the lack of space.

2.2 Symmetries for Mechanical Models

Any model is a simplified version of the real world, where one must specify assumptions in order to capture invariant facts which can be repeated, simulated and generated to improve his execution. The existence of any kind of symmetries allow us to apply replication procedures for different phases, and simplify the distributed design of algorithms. The key fact is that above EMAD gives equivariant stratifications for the representation of groups generated by reflections (Finat, 2000)

In the classical case (rigid bodies, e.g.) constraints are linked to the preservation of scalar quantities in the kinematic (energy functionals) or the dynamic framework (virtual work), or to the preservation of vector quantities (angular momenta) by vector fields, which are considered in an independent way. In locomotion tasks of multibody systems instead of looking at the preservation of above scalar or vector quantities (which is not true), we must look at the preservation of the global structure supporting the dynamics, which is given by a Clifford algebra (a symplectic or a contact structure in the classical case). The specification of the mathematical model is some cumbersome and I shall restrict myself to comment that the emphasis is put on the preservation of global mechanical constraints involving to any prolongation by means invariant differential operators defined onto the jets space.

Indeed, geometric, kinematic and dynamic constraints are also modifying along the locomotion, and there exists a feedback between scalar and (multi)vector constraints; thus, we must introduce a formulation to control the exchange between scalar and vector quantities with changing constraints depending on the phase of the motion. The periodicity conditions about these phases involve also to optimization criteria which must have also present as a periodic or at least an alternating behavior.

Locomotion tasks for biped or multilegged robots involve to the generation of periodic patterns including stance, land-off, navigation and ground impact phases for each leg. The concatenation of all these phases is performed in terms of flexion/extensions operations allowing shape changes in articulated mechanisms. To the lowest level, this agonistic/antagonistic behavior for articulations is modelled in terms of reflection groups. The coupling between components provides the out-of-phase criteria for the reflections associated to the coordinated movements between legs. We interpret the body (including the other leg along the navigation phase) as a flexible plate which contributes to the stability of the global system. Symmetries appearing in gait are globally controlled by polygonal groups (depending on the number of legs and the gait type), which provide a switching process between opossite legs. So, it suffices to analyze the behavior along a complete cycle for each leg and then to apply the action of the hexagonal group to control the distribution process between different legs.

3 ALGORITHMS FOR MOTION PLANNING

The motion planning problem is old and well-known in Robotics. Different approaches to the algorithms design have been developed by using techniques arising from Computational Geometry (roadmap, cell decomposition and potential fields, e.g.) Probabilistic Modeling (local methods including evaluation and inference models), and Artificial Neural Networks (supervised vs. unsupervised learning procedures, self-organising maps) depending on the task and the environment (Latombe, 1991), (Berg and O.Schwarzkopf, 1997). In all cases, one uses different optimization criteria based on least squares method (LSM), maximum likelihood (ML) and genetic algorithms (GAs). The increasing complexity of tasks and the need for adaptability to different environments, require a combination of the above catalogue of techniques and the development of hybrid methods to achieve a better performance. In our case, the locomotion is by itself a enough complex task, and hence the emphasis for the planning is put onto the propioceptive system, rather than in the scene characteristics which we shall suppose free obstacle (otherwise, one could apply the above methods for the center of gravity).

3.1 General remarks about algorihtms for locomotion

The planning and generation of movements for kinematic multichains of changing shapes corresponding to articulated components (arms, legs, body or hands) is a difficult task even in structured scenes with static objects (Wilfong, 1988) is one of the first references for algorithms relative to mobile data. The most difficult problem is linked to the management of different kinds of models and constraints (Canny, 1987). The compatibility between constraints requires the incorporation of mobile data structures and fast updating procedures. The updating involves to the design of mobile kinetic data structures (Guibas, 1998); two neck bottles of this approach arise with the difficulties to an efficient management of spatial data (a common problem of available software in Computational Geometry), and the lack of structural models arising from changing vector fields. There is no a general answer to these problems, still; for the locomotion case in the meantime one can develop good graphic simulations and deduce mechanical models controlled by known vector fields. This is exactly our strategy.

There are several approaches to the algorithms design depending on the framework and the complexity of the task. Some criteria giving crossed classifications are related to the learning processes (supervised and unsupervised), or with model-based character (structural and randomized algorithms), depending on the emphasis onto trial-and-error procedures or the availability of models.

Obviously, the best solution for unstructured scenes will be a hybrid combination of both approaches, but in this note I shall put emphasis on those based on mechanical models and enough information about the scene. Models can be biologically inspired or mechanically based; the biological inspiration provides criteria for the human-based expected behavior, but this gives a coarse approach: each human presents some special gait, and makes difficult the transference of biological principles to control modules. Instead, one can use mechanical models obtained from the Lagrangian's formulation of the motion equations for each leg, which provides a decoupled input to be used in another parts of the process. As always, the best solution would must be a hybrid one, but the integration is not easy to perform it.

Main issues of algorithms for hierarchised models in parallel robots concern to

- Perception: capture, preprocessing and fusion of information processes
- Mechanics: including geometric, kinematic and dynamic aspects of movements
- Motion planning, simulation and evaluation and their symbolic representation in terms of oriented graphs with transition, patching and folding phenomena.
- Adaptive Control: Static, gradient, marginal and dynamic, including an strategy for switching and tuning.
- Execution and monitoring: Generation of impulses, evaluation, comparison between current and desired trajectories, correction of errors.

They are linked between them, and this makes difficult to give a global overview of all of them. Nevertheless, I shall paid my attention about some aspects related to mechanical aspects and motion planning, where structural models emerge in a natural way from the stratified nature of tasks and constraints involving to *articulated mechanisms*. The basic idea of this note concerns to the incorporation of hierarchised models in both processes. In both cases, one has a natural hierarchy which can be expressed in terms of the natural projection $\pi : \mathcal{C} \to \mathcal{W}$ and their successive prolongations $j^k\pi$. So, for $k = 0$ we shall have the topological and geometric aspects, for $k = 1$ the kinematics and for $k = 2$ the dynamics (Finat and S.Urbaneja, 1998). A complex task to be performed by a parallel robot composed by several independent kinematic chains requires a multi-point or a multi-line approach with a strongly hierarchised scheme of switching and tunning processes.

3.2 Minimal remarks about Symmetries for Optimization and Control

It is very important to remark that all these processes must incorporate some kind of symmetries for transmission, co-ordination and control to simplify the distribution of processes, the balance between dynamic effects and the design of accesibility and controllability in locally symmetric terms (Lie algebras) for the control chart.

The algorithm design must incorporate also some evaluation procedure, where one can apply optimization standard procedures to improve the execution of movements. This involves to the choice of objective functions linked to cost functions, and this is not easy for parallel robots, by virtue of their distributed character, and the multiplicity of control elements (points or lines, involving to encoders and actuators, e.g.).

By example, a multiobjective nonlinear programming in parallel or sequential form controlled by different Lyapunov functions (one for each controller which is associated to different phases of a complex task), disregards the different nature of scalar and vector constraints, and the transference of information (mechanical connections) between them. In simplified models, this can be performed in terms of usual Moment Map (Finat, 2000), where one has two kinematic scalars relative to the energy functional and the square of the norm of the angular momentum; both of them are the basic invariants for the mechanics of rigid bodies, but this is not true for the locomotion of articulated robots. Thus, it is necessary to construct vector fields able of explaining how these changing quantities are periodically transferred between different components of articulated mechanism and phases of movement, including sequential phenomena onto spatio-temporal models linked to the dissipation effects (due to friction and impact phenomena) and delays associated to out-of-phase between components. In the meantime, I shall give some ideas which work to low-level from the identification of some elementary symmetric patterns.

3.3 Symmetries for planning

The existence of different kinds of symmetries (rigid movements, e.g.) or alternate periodic processes (translated into switching and tunning) makes easier a modular design for planning. Spatial symmetries can be described (Chasles, hacia 1860) in terms of reflection groups involving to motion at joints and to the description of translations and rotations as composition of reflections (Hestenes, 2000). Next, one can add temporal delays to incorporate alternance or periodic effects onto the artificial mechanism. Thus, I shall restrict myself to situations where symmetries are easily identifiable (gait tasks, e.g.) in terms of reflections. Terrestrial locomotion is not only the capability of self-propulsion, but also the coupling between gravitation and inertia effects in a passive or active way. Such coupling concerns to an exchange between scalar and vector functionals, s.t. the changing energy and angular momenta of each kinematic chain.

Most of industrial robots are composed by only one kinematic chain (the 6R would be a typical one) or by a mobile platform which is stopped before performing another tasks. However, a parallel robot is composed by a finite collection of kinematic chains (legs of a multilegged robot or fingers of an artificial hand) which are connected to a mobile platform (Merlet, 1996), and with an alternating character (open and closed loops), depending on the phase of the task to be performed. This changing character is physically interpreted as a phase transition or as a singularity for some kinematic or dynamic model. The algorithms design for locomotion must evaluate the current state in $\mathcal{W}$ to prevent the alternance between open (allowing translations and rotations in $\mathcal{W}$) and closed (only rotations in $\mathcal{W}$) chains.

Particular solutions can be learned in a heuristic way (s.t. the Honda biped realises currently; see (Hirose and K.Yoneda, 1998) and references therein). The control points located onto articulations describe quasi-periodic curves which display a regular kinematic behavior. These curves are located in a dynamical stability region of the spatio-temporal working space which can be interpreted as a tubular neighborhood with a periodic curve as nerve. The radius of the tube is bounded by the permissible oscillations of the pressure center C_P along the locomotion. The pressure center is the virtual position of the center of gravity C_G under the effect of virtual displacements produced by the resultant of forces and momenta (wrenches) in non-equilibrium positions. In prevision of failures in the kinematic and dynamic data of control points, and due to the double distributed character as hierarchised and parallel system, we must implement a control chart with the corresponding optimization criteria for hyperredundant articulated mechanisms.

Anyway, there are well-known algorithms for transference and fine motions, which operate following a decision tree for each open kinematic chain: one starts by introducing a material point (C_G by example), one constructs feasible trajectories compatible with mechanical constraints, one simulates the behavior of solutions and one optimizes the resulting solutions according to metric, probabilistic or smoothing criteria. This approach is based on the mechanical behavior of a material point (the center of gravity, e.g.) and the construction of a tubular neighborhood for feasible trajectories where the optimization and control is performed. So, the incorporation of additional constraints follows hierarchised scheme which makes easier the updating of the information process in changing environments. To avoid an expensive maintainance in odometric terms ones uses the lines geometry: it incorporates incidence conditions which are computationally translated in doubly connected lists.

4 A MODEL-BASED APPROACH FOR THE ALGORITHMS DESIGN

Following our hierarchised approach, the algorithms design for locomotion planning can be formulated w.r.t. the articular or configuration $\mathcal{C}$ or the working space $\mathcal{W}$. Both spaces are linked by the natural projection $\pi : \mathcal{C} \to \mathcal{W}$, and their k-th order prolongations (see the EMAD, below). The local geometry of the global situation presents a lot of pathologies (singularities, e.g.), and it is convenient to adopt a simplified representation obtained by overimposing onto the working space a general hierarchised combinatorial geometric structure (supported onto flag manifolds) for a changing mechanics, motion planning and an adaptive control of articulated mechanisms in locomotion tasks. The tangent space to this hierarchised system provides enough vector fields to support any exchange between different modules.

Failures between current and desired dynamical data of the certificates associated to the control points or lines can appear along the updating process. It is necessary to prevent and correct these discrepancies in terms of a queue of internal events. The most important problem concerns to the feedback between different levels (geometry, kinematics and dynamics) of mechanics. To this goal, the use of the lines geometry for the support allow us to avoid the odometry problems (the calibration can be restated in these terms, but it will not be considered here by space reasons). Next, one introduces the simplest scheme based on ordinary small rotations in $\mathcal{C}$ and one propagates along the mechanism. The difficult question concerns to optimize the sequencing rotation at joints along each kinematic chain of the multibody for a better performance of motion. This is achieved by using symmetric temporal patterns of the dynamical models with hierarchised control systems (symmetries are generated by reflections); a SVD for the pseudo-inverse matrix associated to several simultaneous rotations allow us to optimize the right distribution of impulses.

4.1 Algorithms and constraints for the Geometry

The geometric aspects of algorithms design depend on the architecture and the scene. Their goal is to determine the reachability region for each component in terms of the maximal elongation of legs (Delaunay triangulations). If all the joints are rotational, this constraint is computed by differentiation of the length constant conditions for each component; the coupling for each leg is performed onto a three-dimensional torus $T^3 = S^1 \times S^1 \times S^1$, where each S^1 corresponds to a rotational joint. The resulting values for each component are patched together and this gives a planar representation in terms of intersecting circles, to characterize feasible equilibrium postures; to do that one uses lists of certificates which evaluate and correct the errors arising from small differences between desired and current position-orientation by using polar coordinates for each rotational joint.

The new aspect concerns now to the feasibility provides effective criteria to describe an average between singular or exceptional configurations for each leg (characterized by $\theta_i = k\pi$ for each rotational joint, e.g.), and admissible values to avoid self-collision between different legs based on allowable circles to be simulated by each leg before starting the motion. The influence region of each leg for each phase in the aerial navigation phase is computed from a mobile planar Voronoi diagram in the working space associated to six mobile points corresponding to the orthogonal projection onto the motion plane of the ends of legs.

In fact we must look only at most three mobile points in swing phase: a) in the biped case we have two triples of aligned points given by the (projections of) of hip, knee and ankle as control nodes for each leg, which gives degenerate Voronoi diagrams (by the alignment of control nodes); b) for more stable static machines s.t. hexapod robots one has a similar reasoning, but by replacing the above control nodes by the ends of each leg.

The convexity constraints for influence or Voronoi regions allow us to apply standard optimization criteria. All these constraints determine optimal equilibrium configurations for the polygonal support (tripod gait at least for multilegged robots) associated to the dual Delaunay triangulation. By using the maps appearing into the EMAD, it is possible to transfer this information to the working space with corresponding constraints g; in addition, the existence of local sections, allows us to lift this information to the kinematics and dynamic framework, and to evaluate the coherence with additional constraints of higher level.

4.2 Algorithms and constraints for Kinematics

Kinematic aspects: They concern to the generation of impulses, the transmission along the truss and the coordination between legs. To simplify the transference problems it is convenient to adopt from the beginning a formulation in terms of twists (lines representing the angular momentum and the linear momentum of a line). This program includes

1) The evaluation of desired values of kinematic variables along marginal stable trajectories. This concerns to the resolution of direct and inverse kinematics given by the Jacobian matrix and its pseudoinverse associated to $j^1\pi$. In this phase one can apply the Pseudo-inverse algorithm (Kerr and B.Roth, 1986). The simultaneous optimization is performed in terms of SVD.

2) The adjustment of values for local invariants of each motion phase following the equivariant version of the EMAD. In this phase, one must use linear programming techniques (simplex method, e.g.) but adapted to multivector quantities (Finat, 2000), and standard non-linear programming for quadratic distance functionals.

3) The selection of a threshold for transmission phenomena involving to the exchange between scalar and vector quantities along the architecture (propagation phenomena onto each leg as a soliton, e.g.). This work in progress must combine recent approaches to the Mechanics with Symmetry (Marsden, 1992) with the implementation of controllers design for changing dynamics (Zefran and J.W.Burdick, 1998)

4) The introduction of nonlinear constraints for a limited coordination between legs (controlled by polygonal groups depending on the parallel robot and the task to be performed). Here, one can apply some adaptation to the Locomotion of the usual Lagrange multipliers method (Nakamura and T.Yoshikawa, 1989).

4.3 Algorithms and constraints for Dynamics

The dynamic aspects concern to the interactions between different components and with the ground. The hyperredundant character of multibody systems gives non-unique solutions for the distribution of forces and moments (grouped as wrenches in a representation in terms of the geometry of lines). Hence, one must to apply different optimization procedures to evaluate right evaluation of wrenches which are meaningful to achieve the locomotion. The reasoning scheme is exactly the same than above for kinematics, but by replacing now the twists by wrenches, with the corresponding interpretation for the dynamical effects which have been disregarded in lower level analysis.

So, a right evaluation of the model would must guarantee an efficient transference between different quantities (even in presence of singularities), to make corrections of impulse generation, switching and tuning processes for the activation/inhibition of movements for each component. The algorithm must evaluate the effect of rotational vector fields acting simultaneously onto several joints in $\mathcal{C}$ to perform a movement in $\mathcal{W}$, and inversely. This involves to the SVD of the matrix expression, again.

Global constraints are introduced in terms of Lagrange multipliers involving to each phase: so, they arise from relative location of interaction forces in friction cones in phase transitions for the ground contact (which involves to the transitions phases in locomotion with an interchange between closed and open loops for kinematic chains in stance and swing phases), and incidence conditions about action lines (representing forces and momenta for each component) for the swing phase. The right coordination requires a model able of integrating both of them. The Clifford Calculus (Hestenes and G.Sobczyk, 1984) provides the theoretical framework to perform this integration (Finat, 2000).

5 CONCLUSIONS AND FUTURE DEVELOPMENTS

In this work, I propose a general model for the Mechanics, Motion Planning and Control of Multilegged Robots, with a biological inspiration arising from the biped humanoid with some restrictive assumptions. This hierarchised model supports an equivariant structure, to simplify the design of control charts, and to allow the reusability from replication of elementary patterns by means of different kinds of reflections. Obviously, this model can be extended to another multilegged or parallel robots, is the next step.

Propagation phenomena are geometrically modeled in terms of reflections. To develop a more dynamical approach, one needs to control them in terms of vector fields. This problems concerns to the interaction with the environment, and the identification of geometric changing elements for the gait analysis. In both cases, the reflections play also an important role, and this presence allow us to develop the same approach. To identify the mirrors, one can adopt some kind of mid point rule for equilibrium postures or G-average for scalar time-dependent functionals. Both strategies appear in recent formulations of analytic mechanics (Marsden, 1992), but as a technical trick more than a systematic approach in a common geometric framework. The main source for errors arises from troubles to estimate changes appearing in phase transitions. One needs to develop spatio-temporal dynamical models to improve the matching of a changing dynamics.

The neck bottle is linked to the treatment of mobile data. The continuity properties of rigid transformations acting onto articulated mechanisms allow us to obtain parcels of integral curves onto the postures spaces, whose properties give certificates to evaluate the correctness of the current trajectory w.r.t. the desired or planned trajectory. One can perform a probabilistic evaluation of certificates based on a proximity notion between current and desired trajectories inside a tubular neighborhood corresponding to a dynamical extension of the usual kinematic marginal stability criteria. The framework for this extension would must be an extension to the 3D case (including dynamic effects) of the Kinetic Data Structures (KDS) (Guibas, 1998).

REFERENCES

Berg, M., M. M. and O.Schwarzkopf, 1997. Computational Geometry. Algorithms and Applications. Springer-Verlag.

Boissonnat, J.D., O. and S.Lazard, 1995. Motion planning of legged robots. In: Algorithmic Foundations of Robotics (WAFR'94), A.K.Peters, pp. 49–68.

Canny, J., 1987. The complexity of Robot Motion Planning. The MIT Press.

Finat, J., 2000. Using perceptual symmetries for monitoring action trajectories (to appear).

Finat, J. and S.Urbaneja, 1998. An integrated framework of kinematic and dynamical aspects for a hexapod robot in jets space. In: CLAWAR'98 (Climbing and Walking Robots) First International Symposium (Brussels, 26-28 November), BSMEE, ASME, pp. 173–178.

Finat, J. and S.Urbaneja, 1999. Dynamical transmission along articulated legs. motion coordination and stability for a hexapod robot. In: CLAWAR'99 (Climbing and Walking Robots) Second International Symposium (Portsmouth, UK), Professional Engineering Publishing, pp. 267–277.

Gonzalo-Tasis, M., J. R. and D.Sanchez, 2000. Symbolic models for postures recognition of a three-fingered artificial hand. In: XIXth Congress of the ISPRS (Amsterdam, July 2000), Int. Soc. for Photogrammetry and Remote Sensing.

Gorce, P., 1998. Dynamic control of bipeds uisng postural adjustment strategy. In: SMC'98 (Systems, Man and Cybernetics, San Diego, October 1998), IEEE.

Guibas, L., 1998. Kinetic data structures: A state of the art report. In: Algorithmic Foundations of Robotics (WAFR'97), A.K.Peters, pp. 191–210.

Hestenes, D., 2000. Old wine in new bottles: a new algebraic framework for computational geometry. In: E.Bayro-Corrochano and G. (eds) (eds), (to appear), Birkhauser Verlag.

Hestenes, D. and G.Sobczyk, 1984. Clifford Algebra and Geometric Calculus, a unified language for Mathematics and Physics. D.Reidel, Dordrecht.

Hirose, S., H. and K.Yoneda, 1998. Normalized energy stability margin: Generalized stability criterion for walking vehicles. In: CLAWAR'98, Climbing and Walking Robots (Brussels, 26-28 Nov, 1998), IEE-ASME-BSMEE, pp. 71–76.

Kerr, J. and B.Roth, 1986. Analysis of multifingered hands. International J. of Robotics Research 4(4), pp. 3–16.

Latombe, J., 1991. Robot Motion Planning. Kluwer Acad. Publishers.

Marsden, J., 1992. Lectures on Mechanics. Cambridge University Press.

Merlet, J., 1996. Les robots paralleles. Hermes.

Nakamura, Y., K. and T.Yoshikawa, 1989. Dynamics and stability in coordination of multiple robots mechanisms. International J. of Robotics Research 8(2), pp. 263–270.

Wilfong, G., 1988. Motion planning in the presence of movable obstacle. In: ACM Symposium on Computational Geometry, ACM.

Zefran, M. and J.W.Burdick, 1998. Stabilization of systems with changing dynamics. In: Workshop on Hybrid Systems: Computation and Control, Berkeley, CA.