LIGHT ABERRATION EFFECT IN HR GEOMETRIC MODEL

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ABSTRACT:
The advent of commercial observation satellites with an increasingly fine resolution requires the definition of whenever more accurate geometrical models to correct most of the main image distortions. This effort must be taken into account earlier in the image processing chain so as to minimize the use of external references, sometimes unavailable and often costly, to correct and improve image geometry of commercial image products.. Sometimes this is only defined by empirical models like 2D/3D polynomial or 3D rational functions supported by most commercial imaging software applications. In others cases those products are supplied with a simplified rigorous model, made up of attitude, ephemeris and camera model information suitable for advanced photogrammetric processing (i.e. ortho-rectification). The Pleiades Perfect Sensor Product will have these two kind of features whose use will depend on customers image processing capabilities. After a brief description of the definition of the processing related to the production of the Pleiades Perfect Sensor Imagery, this paper will focus on the “aberration of light” phenomenon and its incorporation on the raw rigorous geometric model in order to improve its intrinsic accuracy. This problem will be developed in the general framework linked to Earth observations satellite.

1. INTRODUCTION

The advent of commercial observation satellites with an increasingly fine resolution requires the definition of whenever more accurate geometrical models to correct most of the main image distortions. At the same time, the current trend of image resellers is to provide standard imagery in a user-friendly sensor geometry.

A compromise has been found: it consist in defining a simplified sensor model allowing advanced photogrammetric processing and without a loss of accuracy.

After a brief description of the definition of the processing related to the production of the Pleiades Perfect Sensor Imagery, this paper will essentially focus on the “aberration of light” phenomenon and its incorporation on the raw rigorous geometric model in order to improve intrinsic accuracy of derivatives products as perfect-sensor or ortho-images.

At first, this paper will describe the physics of light aberration phenomenon. An analytical formulation of this deviation based on the satellite attitude will be exposed. Then it will evaluate the evolution of this deviation in cases of several kinds of acquisitions (synchronous and asynchronous) and it will give an evaluation of the classic mean calibration residuals (i.e., without taking in account this phenomenon on the raw geometric model).

This residual will be expressed in terms of internal distortions in the image and location performances. Finally, these results will be compared with those obtained with a model taking into account this phenomenon: this will illustrate the usefulness of such calculation

2. PLEIADES PERFECT SENSOR DEFINITION

The Pleiades Perfect Sensor Imagery is the image which would have been got by a perfect push-broom sensor in the same imaging conditions, so it is a virtual raw product.

It is designed for customers having photogrammetric capabilities and who want to exploit the geometric characteristics of the image (DEM or 3D extraction) without having to take into account the complex geometry of the real sensor.

This imagery is thus geometrically corrected from on-board distortions, but not mapped into any cartographic projection.

2.1 Pleiades Perfect Sensor Product Overview

PHR RAW Products will be very complex and not user-friendly, due in particular to the complexity of the detector layout in the focal plane: for instance, the panchromatic focal plane is composed of five slightly tilted arrays.

In order to greatly simplify the use of sensor model, RAW Imagery and data are pre-processed into a standard format model (including RPC coefficients) which will be supported by most commercial imaging software applications.

The geometric reference frame for Perfect Sensor Imagery simulates the imaging geometry of a simple pushbroom linear array, located very close to the panchromatic TDI arrays. Besides, this ideal array is supposed to belong to a perfect instrument with no optical distortion and carried by a platform with no high attitude perturbations. This attitude jitter correction (made with a polynomial fitting) allows both for simple attitude modelling and more accurate representation of the imaging geometry by the rational functions sensor model.

The use of one single Perfect Model, common to all bands (panchromatic and multi-spectral) systematicly provides registered products: PA and multi-spectral Perfect Sensor Imagery are completely super imposable.

The Perfect Sensor Imagery resolution is related to the RAW imagery resolution which varies between 70 cm (at nadir) to 1m (30° off-nadir look angle) for panchromatic products, and 2.8 m (at nadir) to 4m (30° off-nadir look angle) for multi-spectral imagery

2.2 Processing

The production of this ideal linear array imagery is made from RAW Imagery and its most rigorous sensor model. The perfect sensor imagery is then corrected of some geometrical distortions which can be grouped into two general categories: those related to the acquisition system (which includes the orientation and movement of the platform, i.e orbital and attitude disturbances and the sensor optical geometric characteristics, i.e optical distortion, scan distortion, …etc) and those related to the observed object (which takes into account atmosphere refraction and terrain morphology, but also light-aberration phenomenom which will discuss below.

In particular, the registration of panchromatic and multi-spectral bands in perfect sensor imagery is performed taking in account the topographic distortions.

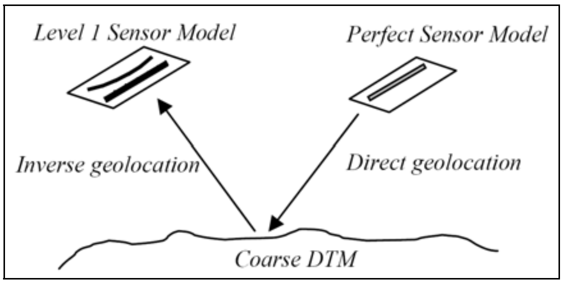

Figure 1. Perfect sensor resampling process

RAW Imagery is resampled into the Perfect Sensor geometry taking a coarse DTM into account. The direct geo-location is made with an accurate Perfect Sensor rigorous model. Thus, Perfect Sensor Imagery and its Perfect Sensor Model are consistent.

The impacts of the above processing on the geometric accuracy of the resulting products have to be significantly small (errors less than centimetres). These errors are due to:

- Direct and inverse location function accuracy,
- The quality of the resampling process and
- The accuracy of the coarse DTM used (generally SRTM, and if not available, Globe. To obtain optimized results:
- Location grid sampling is adequately chosen,
- Resampling process is made with a highly accurate method (using spline interpolators) which not shades off imagery, and
- the DTM is pre-processed in order to minimize the relief artefacts due to errors and/or blunders.
- And the geometric model differences (especially attitude and detector model) between Perfect sensor and RAW sensor are minimized to decrease the parallax and the altitude error effects.

So the Perfect Sensor Model is a compromise between its smoothness and its high similarity to RAW Sensor Model.

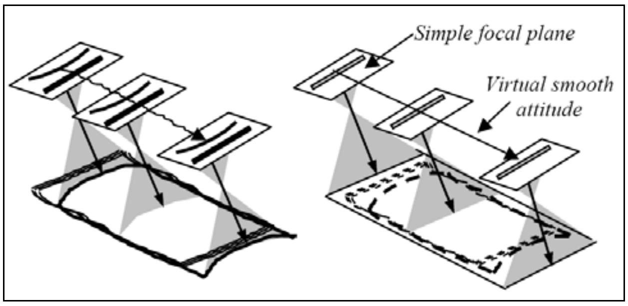

Figure 2. Perfect sensor geometry

3. ABERRATION OF LIGHT PHENOMENOM

3.1 Usual Definition

The aberration of light is an astronomical phenomenon which produces an apparent motion of celestial objects. It was discovered and later explained by the third Astronomer Royal, James Bradley, in 1725, who attributed it to the finite speed of light and the motion of Earth in its orbit around the Sun.

At the instant of any observation of an object, the apparent position of the object (i.e. the direction in which it is seen by an observer on the moving observer frame) is displaced from its true position (i.e. the direction of the straight line between the observer and star at the instant of observation) by an amount which depends upon the transverse component of the velocity of the observer, with respect to the vector of the incoming beam of light: the difference between these two positions is caused mostly by "aberration".

This phenomenon is usually taken account by astronomers when they calculate the pointing of their telescope. In this case the aberration of light is caused by the motion speed of the Earth around the Sun (the maximum amount of the aberrational displacement of a star is approximately 20 arc-seconds in right ascension or declination).

This phenomenon is also classically taken account in stellar sensor processing, which deliver absolute attitude information from stellar observations corrected of this aberration.

3.2 Aberration of light in satellite observation

In the same way that astronomers take into account the motion speed of the Earth around the Sun in the calculation of the pointing of their telescope, the geometric model of image acquisition satellite must take into account this same effect to improve its intrinsic accuracy. Indeed, in the case of Satellite Earth observation, the platform speed is not negligible compared to the light speed, so we must take into account the composition of classical speeds at incident light rays level. A satellite located at an orbit altitude of *700 km*, has a speed about *7 km/s*. This causes an angular deviation of *23 μrad* of the viewing direction corresponding to a ground distance of approximately *20 meters*: this is far from negligible for high resolution imagery.

The deviation angle δ is the angle between the "theoretical" or "real" direction $\vec{u}$ and the "apparent" direction $\vec{w}$.

This deviation is due to the fact that the observer is moving at a significant speed $\vec{V}$ in a direction different from its direction of observation.

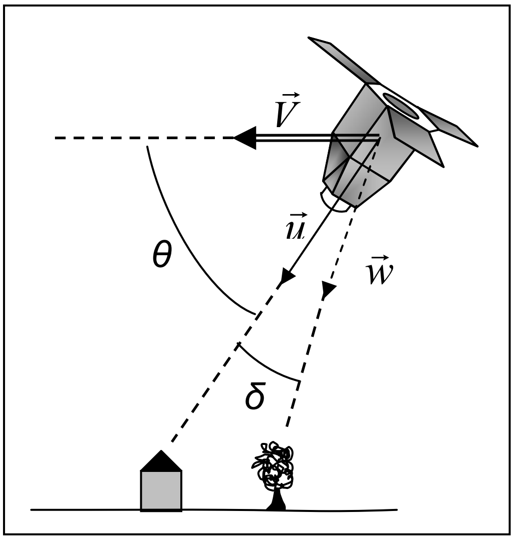

Figure 3. Aberration of light in satellite observation

Thus when the observer points to the direction $\vec{u}$, beam light seems to come from another direction given by $-\vec{w}$. The angle between $\vec{u}$ and $\vec{w}$ sets the angle of deviation δ.

Assume there are two frames *S* (Satellite) and *E* (Earh), each using their own Cartesian coordinate system to measure space and time intervals: *S* uses (t, x, y, z) and *E* uses (t, x', y', z'). The coordinate systems are oriented so that the *x*-axis and the x' -axis overlap, the *y*-axis is parallel to the y' -axis, as are the *z*-axis and the z' -axis. The origins of both coordinate systems are the same.

The relative velocity between the two frames is $\vec{V}$ along the common *x*-axis.

Angle value δ can be deducted from the expression of direction of beam light propagation in the fixed (*E*) and mobile (*S*) frames.

We note $\vec{U} = \begin{cases} U_x \\ U_y \\ U_z \end{cases}$ the direction of the beam light propagation in the mobile frame (satellite frame). We have : $\left\|\vec{U}\right\| = c$.

Assume that $\vec{W}$ is the direction of beam-light in the fixed-frame. Its components are deducted from those of $\vec{U}$ by application of the Lorentz transformation.

$$\vec{W} = \begin{cases} W_x = K \cdot (U_x + V) \\ W_y = K \cdot \beta \cdot U_y \\ W_z = K \cdot \beta \cdot U_z \end{cases} \quad \text{with } \left\|\vec{W}\right\| = c \qquad (1)$$

$$\text{with: } \beta = \sqrt{1 - \frac{V^2}{c^2}} \quad et \quad K = \frac{1}{1 + u_x \cdot \frac{V}{c^2}} \qquad (2)$$

This transformation guarantees the invariance of the magnitude of $\vec{U}$ and $\vec{W}$: the beam-light propagates with a velocity equal to *c* in any frame !

This deviation depends on the angle θ between the direction of motion of the observer $\vec{V}$ and the direction of observation $\vec{U}$: it is maximum at 90°(it is the particular case of Nadir acquisitions) . In this case we have :

$$\vec{U} = \begin{cases} 0 \\ U_y \\ U_z \end{cases} \text{ and } \vec{W} = \begin{cases} V \\ \beta \cdot U_y \\ \beta \cdot U_z \end{cases}$$

The angular deviation δ can be evaluated by the following expression :

$$\sin \delta = \frac{\vec{U} \wedge \vec{W}}{\left\|\vec{U}\right\| \cdot \left\|\vec{W}\right\|} = \frac{V}{c} \qquad (3)$$

We can notice that δ can reach $23\,\mu rad$ for classical observation orbits (*700 km*).

In fact, for magnitudes of $\vec{V}$ lower before *c,* (typically less than *10 km/s*), the Lorentz transformation is not necessary. The calculation of the angle δ can be approximated using a classical Galilean transformation.
We can demonstrate that the contribution of the relativist (Lorentz) formulation is related to $\frac{V^2}{c^2}$, so insignificant for our applications.

3.3 Satellite observation General application

3.3.1 Expression of the Image acquisition velocity

As seen previously, the calculation of the angular deviation needs to express the relative speed between the observer (pixel) and the observed object (to the Earth's surface).

Assume that *C* is the centre of the Earth, G is the centre of gravity of the satellite and *M* is the pixel image located in the focal plane of the optical instrument.

We can decompose the motion of *M* ($\vec{V}(M)_{/T}$ with respect to the earth frame) into 3 terms of different origins:

$$\vec{V}(M)_{/T} = \underbrace{\vec{\omega}_{Earth} \wedge \overrightarrow{CM}}_{1} + \underbrace{\vec{\omega}_{Sat} \wedge \overrightarrow{CM}}_{2} + \underbrace{\vec{\omega}_{Inst} \wedge \overrightarrow{GM}}_{3} \quad (4)$$

The first term refers to the earth rotation: For an orbit at *700km*, it corresponds approximately to $500 \ \ m/s$

The second term refers to the motion of the local orbital frame with respect to an (pseudo)-inertial frame : it corresponds approximately to a speed of $7400 \ \ m/s$

Finally, the third term refers to the movement of the camera in its orbital local frame: it corresponds to the steering of the satellite (or instrument) which allow the acquisition of images with a specific guidance. The advent of satellites more manoeuvrable and agile, allowing image acquisitions in very different conditions like cross-track acquisitions (the direction of acquisition is perpendicular to the natural platform motion), high off-nadir acquisitions, or asynchronous acquisitions (with an guidance causing a slowing motion of the line of sight) makes this term less and less negligible. In fact for a satellite with an agility less than *10°/s* and a dimension size less than *5 m*, this term is relatively low, approximately to *1 m/s,* so it is insignificant for the δ calculation.

The components of $\vec{V}(M)_{/T}$ in local orbital frame are then:

$$\vec{V}(M)_{/T} \approx -D \cdot \begin{bmatrix} \omega_{Earth} \cdot \cos(i) - \omega_{Sat} \\ \omega_{Earth} \cdot \cos(\alpha) \cdot \sin(i) \\ 0 \end{bmatrix}_{OL} \approx \begin{bmatrix} V_X \\ V_Y \\ 0 \end{bmatrix}_{OL} \quad (5)$$

where D is the distance between object (earth) and observer (satellite), i the orbit inclination and α the angular position of satellite on its orbit (a null α corresponds to an equatorial position).

In case of classical earth observation applications, the orbit inclination is almost polar, i.e $i \approx 90°$.

The V_X X-component of $\vec{V}(M)_{/T}$ is along track (i.e parallel to the natural satellite motion), it is constant.

$(V_X \approx -D \cdot \omega_{Sat} \approx 7400 \ m/s)$

The V_Y Y-component is cross-track (orthogonal to the natural satellite motion). Its variation is related to the position in orbit α.

$(V_Y \approx -D \cdot \omega_{Earth} \cdot \cos(\alpha) \approx 500 \cdot \cos(\alpha) \ m/s)$

3.3.2 Deviation angle calculation

The viewing direction of the instrument can be decomposed into 2 components (ψ_x, ψ_y) as illustrated in the *figure 4*, respectively roll and pitch orbital-local angles.

The direction of the beam light propagation in the mobile frame $\vec{U}$ can be expressed in the local orbital frame depending on this pair of angles

$$\vec{U} = -c \cdot \left\| \begin{matrix} \tan(\psi_y) \\ -\tan(\psi_x) \\ 1 \end{matrix} \right\|_{OL} = -c \cdot f(\psi_x, \psi_y) \cdot \begin{bmatrix} \tan(\psi_y) \\ -\tan(\psi_x) \\ 1 \end{bmatrix}_{OL} \quad (6)$$

$$\text{where} \quad f(\psi_x, \psi_y) = \frac{1}{\sqrt{1 + \tan^2(\psi_x) + \tan^2(\psi_y)}} \cdot \quad (7)$$

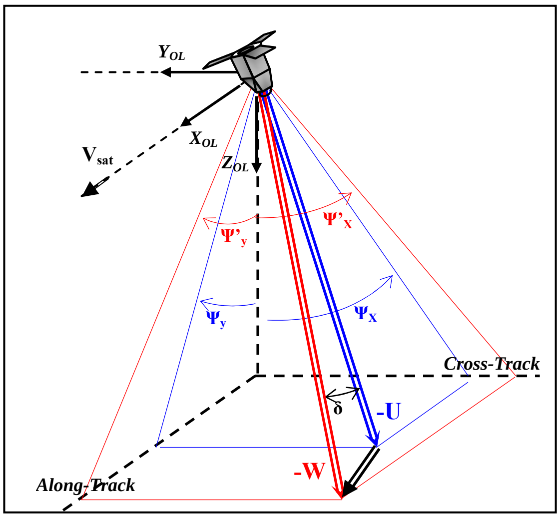

Figure 4. Pointing angles definition

The « apparent » direction beam-light propagation $\vec{W}$ can be written as :

$$\vec{W} = \vec{U} + \vec{V}(M)_{/T} = -c \cdot \left\| \begin{matrix} \tan(\psi_y) \\ -\tan(\psi_x) \\ 1 \end{matrix} \right\|_{OL} + \begin{bmatrix} V_X \\ V_Y \\ 0 \end{bmatrix}_{OL} \quad (8)$$

which can be also be expressed which angles (ψ'_x, ψ'_y) :

$$\vec{W} \approx -c \cdot \left\| \begin{matrix} \tan(\psi'_y) \\ -\tan(\psi'_x) \\ 1 \end{matrix} \right\|_{OL} , \quad (9)$$

As seen at §3.2, we can use a classical Galilean transformation because $\|V\| << c$. In that case $\|W\| \geq c$ but this is without consequence in evaluation of δ .

Magnitude of the deviation angle δ can be easily calculated by following expression:

$$|\sin \delta|^2 = \frac{\|\vec{U} \wedge \vec{W}\|^2}{\|\vec{U}\|^2 \cdot \|\vec{W}\|^2} \approx \frac{\|\vec{U} \wedge \vec{V}\|^2}{c^4} \quad (10)$$

hence,

$$|\sin\delta|^2 \approx \frac{V_X^2 + V_Y^2 + \left(V_Y \cdot \tan(\psi_y) + V_X \cdot \tan(\psi_x)\right)^2}{c^2 \cdot \left(1 + \tan^2(\psi_x) + \tan^2(\psi_y)\right)} \quad (11)$$

The *figure 5* below give the δ dependency towards (ψ_x, ψ_y) pointing angles.

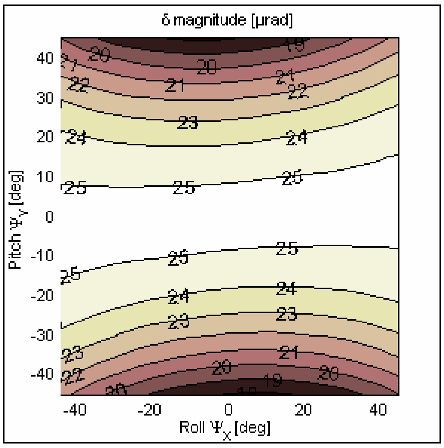

Figure 5. Magnitude of δ vs (ψ_x, ψ_y) angles

This figure shows that the maximum of angular deviation is reached for nadir acquisitions $(\psi_x, \psi_y = 0)$: δ is then equal to *25 μrad* that corresponds to approximately *25 meters* on ground.

Given that $V_X >> V_Y$, δ essentially varies with pitch angle ψ_y.

This result is available for any image acquisition guidance because the contribution of this particular dynamic in the calculation of the speed motion of the camera is considered as insignificant.

4. CALIBRATION OF THE PHENOMENOM

Figure 5 shows that the light-aberration phenomenon causes a deviation of the camera pointing direction which essentially depends on pitch angle. This deviation may cause an error of the absolute location of image acquisition.

The ground errors are plotted on figure 6. The calculation of the along-track and cross-track components takes in account geometric effect of inclined projection on an ellipsoid. That explains why, unlike angular deviation, ground errors in meters increase with pointing angles (values obtained in high off-Nadir angles are higher than those obtained at nadir).

This effect can be calibrated in the geometric rigorous model by introducing a "corrective" rotation at camera orientation level.

Assume that $\vec{\Re}(\delta)$ is the rotation that transforms $\vec{U}$ into $\vec{W}$. This rotation can be written as :

$$\vec{\Re} \approx \frac{\vec{U} \wedge \vec{W}}{\|\vec{U}\| \cdot \|\vec{W}\|} \approx \frac{\vec{U} \wedge (\vec{U} + \vec{V})}{\|\vec{U}\| \cdot \|\vec{W}\|} \approx \frac{\vec{U} \wedge \vec{V}}{c^2} \quad (12)$$

hence,

$$[\vec{\Re}]_{OL} = \frac{f(\psi_x, \psi_y)}{c} \cdot \begin{bmatrix} -V_Y \\ V_X \\ \tan(\psi_y) \cdot V_Y + \tan(\psi_x) \cdot V_X \end{bmatrix}_{OL} \approx \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \quad (13)$$

$$\text{with } f(\psi_x, \psi_y) = \frac{1}{\sqrt{1 + \tan^2(\psi_x) + \tan^2(\psi_y)}} . \quad (14)$$

(r_x, r_y, r_z) are orbital local components of the rotation $\vec{\Re}(\delta)$.

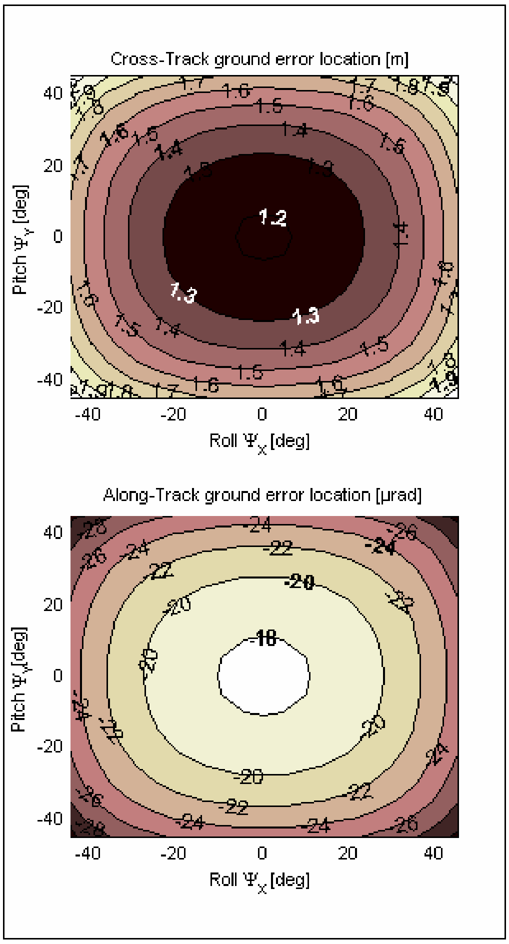

Figure 6. Ground error location components before mean calibration

Only r_x and r_z components have a $\cos(\alpha)$ dependency. However, variations due to *α* are quite low (absolute magnitude is lower than $\frac{V_Y}{c}$ i.e 1.7 μrad approximately). Then we can considerate that these components are *α* invariant.

Magnitudes of (r_x, r_y, r_z) in a large domain $|\psi_x| \leq 45°$ and $|\psi_y| \leq 45°$) are given in the table below

	max	*min*	*mean*	*Error max*
r_x	***1.7***	***0.33***	***1***	***0.7***
r_y	***-14.4***	***-25.0***	***-20***	***5***
r_z	***17.7***	***-17.7***	***0***	***17.7***

Table 1. Max-Min values of (r_x, r_y, r_z)

A mean-calibration processing consists in calibrate rigorous geometric model with (r_x, r_y, r_z) mean constant values.

The *figure 7* below shows the ground residual location error after a mean calibration. We notice that error has considerably decreased with respect to values without calibration (see *figure 6*), but it remains important residuals in $|\psi_x|$ high values domain that can reach *10 meters* on ground.

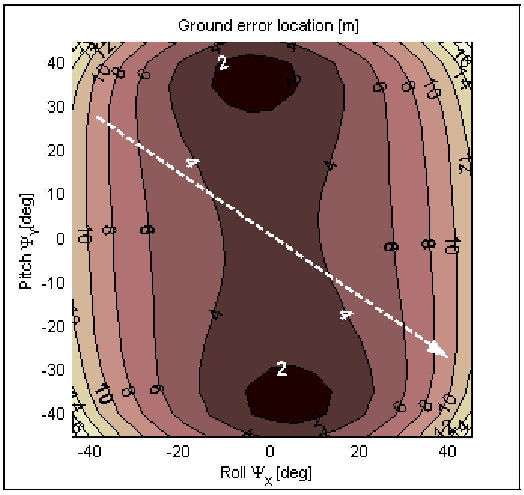

Figure 7. Ground error location after mean calibration

Besides, when image acquisition is performed with "atypical" guidance, pointing angles (ψ_x, ψ_y) may considerably vary during acquisition time.

The white arrow plotted on *figure 7* shows the evolution of (ψ_x, ψ_y) angles during an acquisition of a *350 km* long cross-tack image. We can see that the ground error location residuals vary in a non-linear manner.

5. CONCLUSION

The light-aberration effect has an impact on the calculation of the true instrument direction pointing. In nadir acquisitions, this effect causes a bias, essentially along-track direction, of approximately *25 meters* (for a polar orbit of *700 km*).

If this phenomenon is calibrated by a mean value during commissioning phase, it remains residuals errors, due in particular that the deviation depends on the acquisition pointing angles. These residuals can easily reach *10-12 meters* at ground in very high off-nadir acquisitions (i.e. $\psi_x, \psi_y \approx 45°$).

Besides, with high manoeuvrability satellites, allowing image acquisitions in very different conditions like cross-track acquisitions (the direction of acquisition is perpendicular to the natural platform motion), high off-nadir acquisitions, or asynchronous acquisitions, pointing angles may vary significantly during the acquisition time: the deviation angle δ fluctuates significantly and may cause internal non-linear low frequencies distortions in image.

So, it is more interesting to take into account this effect earlier in the image processing chain so as to minimize these residuals errors, in particular during the elaboration of higher level products like Perfect Sensor or Ortho-images: A earlier rigorous calibration may improve absolute location accuracy of those products of about *5-10 meters* on ground.

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