STUDY OF IMAGE MAGNIFICATION BASED ON CURVELET TRANSFORMATION

CHEN Chao，JIANG Tao

Geomatics College，ShanDong University of Science and Technology,ShanDong Qingdao 266510,China - ayang198206@163.com,tjiang@126.com

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ABSTRACT:

Curvelet transformation proposed in 1999, on the basis of wavelet transformation, is a new multi-scale geometric analysis method. Besides scale parameter and location parameter, it adds an azimuth parameter in structure elements compared with wavelet transformation, which makes the Curvelet transformation express anisotropic singular boundary of lines or curves. So, Curvelet transformation is better than wavelet in geometric features of image, building its basis theory in image magnification. Inverting Curvelet transformation according to the new efficient, the magnified image can be gotten. By designing simulation, the original image is zoomed out to its 2,3,4,6 and 8 times, using cubic convolution, Curvelet transformation and bilinear interpolation. Compared to the original image, the results are showed as follows: whether from the qualitative or quantitative, its effect on image magnification is superior to the cubic convolution and bilinear interpolation.

1. INTRODUCTION

As a basic image processing operation, image magnification is the re-sampling of the two-dimensional data, that is to say, to change the image resolution. Image magnification has been extensively used in image processing, such as image display, analysis and registration,etc.

The early methods of image magnification mainly included nearest neighbor,bilinear interpolation, and cubic convolution. Subsequently,in 1990, Durand et al. presented a method,which interpolates the pixels of digital image gridding as a –spline,and products new digital image by re-sampling new pixels of gridding on zoom requirements. In 1995, Unser et al. developed another method that interpolates the pixels of digital image gridding as a B-spline curve surface,then constructed new Polynomial Spline with the smallest error conditions from the original and target images, and gave the optimal algorithm. Afterwards,in 1997,Darwish et al. presented a algorithm based on adaptive re-sampling, which especially fits for bigger enlarge factor. Using linear interpolation and gradient calculation, Albiol et al. put forward an image magnification technology based on mathematical morphology; the region where the pixels changes slowly linear method interpolate is used and special processing is used for the boundaries. In 1998, Lee et al. presented higher-order spline scaling method, using oblique projection operator to construct simple and rapid image scaling algorithm. And in 1999, Qingjie Sun et al. put forward the magnification method based on Bezier curve surface. In 2000, Leu magnified images by using step edge model and gained more effective magnification effect than that by nearest neighborhood and bilinear interpolation method. Using wavelet decomposition,Wang Wei et al. magnified images and reserved the structure information. In 2001, Muhoz et al. presented scaling algorithm which applicable to image reduction and the computational complexity independent of scaling factor. Malgouyres proposed the non-linear variation question numerical method of image scaling in the same year, which is better to maintain the boundaries. Panda proposed the generalized B-spline interpolation and approximation technology (continuous modeling) of image. Using multi-neural network, Sekiwa overcame the distortion phenomenon occurred in the magnifying process by using general neural network(especially when magnifying multiple is large). Len proposed the ramp edge model which magnifies images and preserves the continuity and definition of original image.

Many of above-mentioned methods, because of computing less, simple, have been widely used. However, without special treatment directed at the edge of the image and texture characteristics in the interpolation process, the enlarged images are difficult to maintain clear boundaries and distinct contours. From the frequency domain, some functions used in image magnification perform as the low-pass filters,which make that the filtered images are prone to loss high frequency components, resulting in jagged edge and fuzzy details of high-frequency.

2. RIDGELET TRANSFORM AND CURVELET TRANSFORM

2.1 Ridgelet Transform

If there is function ψ which satisfies the following condition

$$K_\psi = \int \frac{|\hat{\psi}(\omega)|}{|\omega|^d} d\omega < \infty \qquad (1)$$

where $\hat{\psi}(\omega)$ is the Fourier transform of function ψ. The ridge function $\psi_{a,\theta,b}$ deprived from function ψ which is satisfied the expression 1, is claimed Ridgelet.

$$\psi_{a,\theta,b} = a^{-\frac{1}{2}} \psi\left(\frac{x\cos\theta + y\sin\theta - b}{a}\right) \qquad (2)$$

where a =the scale of the Ridgelet,
u = the direction of the Ridgelet
b =the location of the Ridgelet

By definition, we can see the supporting set of ridge function is the zonal region:

$$\{(x,y)\in R^d \mid x\cos\theta + y\sin\theta \mid -b \le a\}.$$

It is obvious that the ridge function is a constant in the direction of $x\cos\theta + y\sin\theta = c$ and a wavelet function in the vertical direction. Thus, the Ridgelet function adds the direction information along the ridge line on the basis of wavelet. Consequently, the directional linear singularities can be expressed and detected effectively.

For any square integrable function f, the coefficient of Ridgelet transform is:

$$R_f(a,\theta,b) = < f, \psi_{a,\theta,b} > = \int f(x)\bar{\psi}_{a,\theta,b}(x)dx \quad (3)$$

where $\bar{\psi}$ is the plural conjugation of ψ. The reconstruct expression is as follows:

$$f = \int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty} R_f(a,\theta,b)\psi_{a,\theta,b}(x)\frac{da}{a^3}\frac{d\theta}{4\pi} \quad (4)$$

The biggest difference between Wavelet and Ridgelet is the directivity of the ridge wave, that is to say, the angle parameter θ which is the token of the direction is introduced besides the scale and location parameters.

2.2 Curvelet Transform and Application in Image Processing

Curvelet Transform is constructed on the basis of the single scale Ridgelet transform or partial Ridgelet Transform,which is used to describe the object with the singular curve border. Curvelet integrates the merits of that Ridgelet is good at express linear features and Wavelet is good at express point characteristics.

Essentially, Curvelet is a multi-scale partial Ridgelet. And its basic idea is:firstly,disposing the signal with Wavelet transform and decomposing it into a series of different scales sub-band signal, and then, disposing each sub band with Ridgelet transform. The size of the sub-block in partial Ridgelet transform can be different from each other due to the scale. Specifically, the Curvelet disassembly includes the following main steps (Figure1)

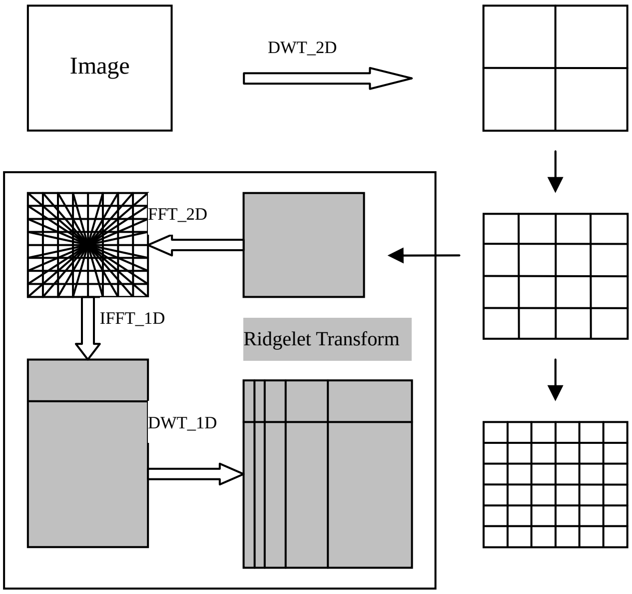
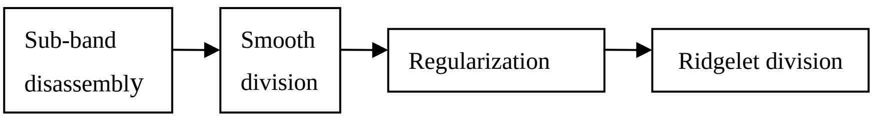


Figure1 Sketch of Curvelet Transform

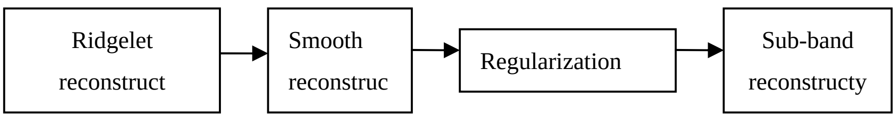
(1) Sub-band disassembly. The image is disassembled into a number of sub-band components by wavelet transform;
(2) Smooth division. The sub-bands are divided into some sub-blocks by Smooth division and the size of each sub-block can be different according to the specific needs;
(3) Regularization. Change the discrete coordinate to the consecutive, and make the continuous supporting region is $[0,1]^2$;
(4) Ridgelet division. Do partial transform with Ridgelet transform for the sub-blocks after the division.

The division and synthesis process of Curvelet transform is shown as Figure2.

The Curvelet transform is presented by Donoho in allusion to the Wavelet isotropic limitation in the expression for the two-dimensional image, whose anisotropy is very helpful to the efficient expression of the edge information. The Curvelet Transform is highly focused by the investigators due to this feature since 1999 and widely used in image denoising, image enhancement, image fusion, image restoration,etc.



Process of Curvelet Transform division



Process of Curvelet Transform reconstruct

Fig2.Process of Curvelet Transform

3. IMAGE MAGNIFICATION BASED ON CURVELET TRANSFORM

In the clear image, the obvious features, such as edge, contour, region,etc, often manifest as gray values and its change in the space domain. After analyzing the conception and algorithm process of Curvelet, we can magnify the image using Curvelet. Curvelet transform coefficient is enhanced on the basis of certain criteria, to increase the contrast between the ridge-line and vale-line, reduce false information. And then the magnified image with clear texture is gained by Curvelet inverse transform.

Curvelet Transform divides the image level into three parts: Coarse, Detail, Fine. From the frequency distribution, low-frequency coefficients (the most lining) are distribute to the Coarse part, high-frequency coefficients (the outer) are distribute to the Fine part, and the middle layer are distribute to Detail. The detail information of clear image is generally included in the high-frequency coefficients of Curvelet Transform.

Using general interpolation method to magnify the image, it will have high precision in the local, but the overall information will be lost and appear the fuzzy edge phenomenon. Using Curvelet transform to magnify the image can improve this.

Image magnification process using Curvelet Transform is as follows:

(1) Using simple interpolation algorithm, magnify the original image f to a certain multiple $expand(f)$.

(2) Having the airspace high-pass filtering of $expand(f)$, getting the high-frequency component $Hexpand(f)$.

(3) Having Curvelet Transform of $expand(f)$ and $Hexpand(f)$ respectively, getting transform coefficients with the same scale.

(4) Adjusting the two group Curvelet transform coefficients using a certain rule. Taking the Curvelet transform coefficients, which have the same resolution level and same location, in the form of addition and weight and a new group of coefficients are gained.

(5) Having Curvelet inverse transform using the coefficients gained from(4),getting the clearly magnified intermediate result image .

(6) Having edge processing of the intermediate result image gained from (5), and the edge of $expand(f)$ is copied to the corresponding edge location in the intermediate result image by the method of pixels copy. Then the final image is gotten.

4. EXPERIMENTAL RESULTS AND ANALYSIS

Simulation experiments are designed based on the magnification algorithm presented in this paper. The original image used in the experiment is Lena image (shown as Figure3 a), size of 50 × 50. The image is magnified by three times by using quadratic linear method (Figure3 b), cubic convolution (Figure3 c) and the new method presented in this paper (Figure3 d) respectively. The magnification effects are compared.

(a)The original image

(b) quadratic linear method

(c) cubic convolution

(d) method in paper

Figure3 Experimental original image and result image

Times	Magnification methods		
	bilinear interpolation	cubic convolution	This paper
2	10.5934	12.3941	16.5681
3	7.8798	8.4121	12.1327
4	5.6013	6.3447	7.7474
6	3.8209	4.2624	5.1029
8	2.9032	3.2203	3.8247

Table1 The average gradient of different magnification methods

Through the comparison with Figure3(d), Figure3(b) and Figure3(c), it is easy to find the magnification image using Curvelet transform has better visual effects, which reduce the fuzzy degree, strengthen the edge detail information, and increase the contrast of ridge-line and valley-line.

From the qualitative aspects just, above, the method in this paper better than the other two methods. Following,we will do quantitative analysis of the experimental results, magnifying the original image by two times, three times, four times, six times and eight times by the three methods mentioned above. And the magnification effects are studied by average gradient.

The average gradient reflects the small contrast, changes of texture and definition in the image. Average gradient is shown as follows:

$$\bar{T} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\sqrt{(\Delta I_x^2 + \Delta I_y^2)/2} \qquad (5)$$

Here M and N express the size of image f .where

$$\Delta I_x = f(i+1, j) - f(i, j) \qquad (6)$$

$$\Delta I_y = f(i, j+1) - f(i, j) \qquad (7)$$

Average gradient of the magnified image is shown in the following table.

From the average gradient in table.1,it can be seen that the image, gotten by Curvelet transform, has higher definition and average gradient, which is same to that by qualitative analysis. However, the average gradients of quadratic linearity and cubic convolution are relatively small, the definitions relatively poor, thus it can be seen, using Curvelet transform to magnify image can get better results.

5. CONCLUSION

According to the needs and characteristics of image magnification, the method based on Curvelet transform is presented and researched in the paper. Magnification experiments are carried out, also compared with quadratic linearity and cubic convolution which are in common use. The validate results show that using Curvelet transform to magnify image gains more clear results, which increase the contrast of ridge-line and valley-line, strengthen the edge detail information, reduce the loss of information as can as possible, and preferably retain the structure information of the original image. The magnification effect is better than quadratic linearity, cubic convolution, and other methods.

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