AN OPTIMISED CELLULAR AUTOMATA MODEL BASED ON ADAPTIVE GENETIC ALGORITHM FOR URBAN GROWTH SIMULATION

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ABSTRACT:

This paper presents an improved cellular automata (CA) model optimized using an adaptive genetic algorithm (AGA) to simulate the spatio-temporal process of urban growth. The AGA technique can be used to optimize the transition rules of the CA model defined through conventional methods such as logistic regression approach, resulting in higher simulation efficiency and improved results. Application of the AGA-CA model in Shanghai's Jiading District, Eastern China demonstrates that the model was able to generate reasonable representation of urban growth even with limited input data in defining its transition rules. The research shows that AGA technique can be integrated within a conventional CA based urban simulation model to improve human understanding on urban dynamics.

1. INTRODUCTION

There is a long-standing interest in understanding urban dynamics though development and application of geographical models (Batty and Xie 1994; Batty, Xie, and Sun 1999; Couclelis 1997; He, *et al.* 2006; Li and Yeh 2002a, b; Liu and Phinn 2003; Muzy *et al.* 2008; Wu 1998, 2002). Compared to many modelling approaches that were developed based on exclusive use of certain mathematical formula, models based on cellular automata (CA) have strong power to capture the non-linear, spatial and stochastic processes of urban growth in more realistic ways (Liu 2008; Stevens, Dragicevic and Rothley 2007; White and Engelen 1993).

Conventionally, CA based urban models require strict definition of various spatial variables and parameters representing different spatial and non-spatial factors driving the development of urban growth (Li, He and Liu 2009). Many CA models have been developed using a diverse range of methods to define such variables and parameters; these methods include multi-criteria evaluation, logistic regression, principal component analysis, and partial least squares regression methods, to name a few. However, limitations of such methods in defining suitable transition rules, or the values of relevant parameters of the transition rules, or in constructing the architecture of the models have been identified and reported in the literature (Al-kheder, Wang and Shan 2008; Li and Yeh 2002a). As a result, there are significant differences between the simulation results and the actual patterns of urban growth, making such models less effective in simulating the actual process of urban growth (Li and Yeh 2002b; Liu and Phinn 2003).

The development of genetic algorithm (GA) and adaptive genetic algorithm (AGA) methods have provided researchers with new ways to identify and search for suitable transition rules and their defining parameters in urban modelling (Bies, *et al.* 2006; Srinivas and Patnaik 1994). This method has been used in satellite imagery classification (Huang *et al.* 2007), site selection (Li, He and Liu 2009), and problem clustering (Lorena and Furtado 2001).

This paper presents a method applying an adaptive genetic algorithm to define and search for transition rules and parameters of a cellular automata model to simulate the spatio-temporal processes of urban growth. The following section presents a generic CA model based on logistic regression method first, followed by the adaptive genetic algorithm method to optimize CA parameters based on minimizing differences between the simulated results and the actual urban development. Section 3 applies the AGA-CA model to simulate the urban growth of Shanghai's Jiading District, Eastern China. Results from the model are also presented and discussed in this section, followed by conclusions in the last section.

2. THE ADAPTIVE GENETIC ALGORITHM BASED CA MODEL (AGA-CA)

2.1 A generic CA model based on logistic regression

Generally, CA defines the state of a cell at one time as a function of the state of the cell and its neighbourhoods at a previous time in accordance with a set of transition rules, which can be generalized as follows (Wu 1998):

$$S_{ij}^{t+1} = f(S_{ij}^{t+1}, \Omega_{ij}, con()) \tag{1}$$

where $S_{ij}{}^{t}$ and $S_{ij}{}^{t+1}$ represent the states of a cell at location *ij* at time *t* and *t*+1 respectively; Ω_{ij} is a neighbourhood function; $con()$ defines a set of constraints or factors affecting the transition of cell states; and *f* is the transition function.

Assume that a cell can take one of only two states, urban or non-urban. A square neighbourhood is defined with u×u cells and all cells within the neighbourhood have equal opportunity for development. Thus, the probability a cell changes its state from non-urban to urban can be defined as:

$$P_{\Omega} = \frac{\sum_{u\times u} S_{ij} = Urban}{u\times u-1} \tag{2}$$

where P_Ω is the probability a cell can change from one state to another; $\sum_{u\times u} S_{ij} = Urban$ represents the total number of urban cells within the u×u cells neighbourhood.

However, in practice, not all cells have equal opportunity for development. For instance, some non-urban areas such as large-scale water bodies or areas with critical physical constraints such as very steep slope may not be able to develop into urban areas. Other areas such as the primary farmland may be prevented from urban development through institutional control, i.e., land use planning regulation.

In order to represent the unequal opportunity of cells for urban development, a stochastic factor can be introduced into the CA based urban models (Wu 2002). With a stochastic control factor, the probability a cell converts from non-urban to urban state can be defined as:

$$P_G^t = \frac{1}{1+\exp(-(a_1 x_1 + ... + a_m x_m))} \times P_\Omega \times con(cell_{ij}^t = suitable) \times (1 + (-\ln\gamma)^\beta) \quad (3)$$

where

- P_G^t is the probability of the cell converting from one state to another at time *t;*
- $con(cell_{ij}^t = suitable)$ is a constraint function, the value of which ranges from 0 to 1, with 0 meaning the cell is constrained from changing its current state, and 1 meaning the cell is able to change its state at the following time step;
- $1+(-\ln\gamma)^\beta$ represents a stochastic factor, where γ is a random real number ranging from 0 to 1, and β is a parameter controlling the effect of the stochastic factor. The value of β ranges from 0 to 10;
- x_i (i = 1, 2, … , m) are various spatial driving factors to urban growth, which can be represented by the distances from a cell to urban centres, town centres, main roads, and so on. These distance factors are also called spatial variables or independent variables; and
- a_1, a_2,…, a_m are used to assign different weight to each of the distance variables.

One of the common challenges in developing a logistic regression based CA model is how to choose the distance variables and configure the relevant parameters defining the impact of such distance factors on urban growth. Consequently, results generated by a logistic based CA model may show poor match for the actual patterns of urban growth. This indicates that there is a need to search for other techniques in identifying and defining the model's transition rules. Such a challenge in model development can be addressed by incorporating the adaptive genetic algorithm approach to randomly search for an optimized conversion probability for each cell, and subsequently minimize the differences between simulated results and actual urban growth patterns.

2.2 Adaptive Genetic Algorithm (AGA) based CA Modelling

A genetic algorithm (GA) is a search technique used in computing to find solutions to optimization and search problems. Inspired by evolutionary biology, genetic algorithms work in computer simulations to search for an exact or approximate solution from a population of solutions (Liao *et al.* 2008). This search and optimization process is achieved according to natural selection, including inheritance, selection, crossover and mutation.

There are two elements of a genetic algorithm, including a genetic representation of the solution domain, such as an array of cells in a cellular urban space, and a fitness function to evaluate and quantify the optimality of a solution.

The efficiency of a standard GA depends largely on the setting of its parameters such as the selection, crossover and mutation rates, which are difficult to adjust manually. Such difficulties can be overcome by the adaptive genetic algorithm (AGA) as the AGA could dynamically modify the parameters of the genetic algorithm (Espinoza, Minsker and Goldberg 2001; Kee Airey and Cye 2001). AGAs not only keep population diversity effectively but also improve the performance of local and premature convergences. Such genetic diversity is important to ensure the existence of all possible solutions in the solution domain and the identification of optimized solution. In addition, the adaptive genetic algorithm also enhances the search speed and precision of the genetic algorithm. Hence, the searching and optimization process for problem solutions can be accelerated.

2.2.1 Fitness function: A fitness function is an objective function to quantify the optimality of a solution. This function was created by selecting sample of cells within the cellular urban space to minimise the differences between the simulation results produced by a logistic regression based CA model and the actual urban growth patterns identified from remotely sensed images. The fitness function is defined and optimised through the modelling process as:

$$f(x) = \sum_{i=1}^{n} (f_i(x_1, x_2, ..., x_m) - f_i^0(x_1, x_2, ..., x_m))^2 \quad (4)$$

where

- $f(x)$ is a fitness function;
- *n* is the number of samples selected from the cell space which were used to retrieve the CA transition rules;
- f_i is the conversion probability of the state of cell *i* based on the logistic regression model, i.e., $f_i = P_G$ as defined in Equation (3); and
- f_i^0 is the actual conversion decision of cell *i*. f_i^0 can only take one of the two values, 0 or 1, with $f_i^0 = 0$ meaning the state of the cell *i* remains as non-urban and $f_i^0 = 1$ meaning the state of the cell has changed from non-urban to urban.

The process of urban growth can be affected by many factors, including socio-economic, physical and environmental, as well as institutional control factors. These factors can be built into the cellular automata model through a set of transition rules. With the fitness function, the simulation process of urban growth can be calibrated by dynamically updating the various parameters of the transition rules to minimise the value of the fitness function so the simulated urban patterns can better match with observed patterns of urban growth. The model calibration process is completed once the fitness function reaches a stable value over time and the model's transition rules and parameters can be considered suitable for operation to the whole cell space.

2.2.2 Coding of chromosomes: Chromosomes are the abstract representations of candidate solutions, which can also be called individuals. A chromosome is a set of parameters which define a proposed solution to the problem that the GA is trying to solve. In the CA based urban modeling practice, all possible CA transition rules factors affecting urban growth are considered as chromosomes. Each chromosome is coded as a simple string like:

$$C = [\alpha_1^k, \alpha_2^k, ..., \alpha_m^k] \tag{5}$$

where C represents a string of candidate solutions; m is the number of spatial driving factors (as in Equation (3)); k means the k^{th} individual (candidate solution). α_1^k to α_m^k represent the weight of each spatial driving factor in the k^{th} candidate solution. In fact, the values α_1^k to α_m^k in the optimized candidate solution are the parameter values required by the CA model as defined in Equation (3).

Initially a number of chromosomes were randomly generated to form the possible solutions for the adaptive genetic algorithm to begin its searching process. After many generations of selection, crossover and mutation operations, only those chromosomes which acquire lower fitness values will remain, resulting in the emergence of a good chromosome structure.

2.2.3 Selection operator: Selection is the key operation of the AGA method in which individual genomes are chosen from a population of candidate solutions for later breeding, including recombination and crossover. During each successive generation, individual solutions are selected through a fitness-based process, where solutions with lower fitness values are typically more likely to be selected. Using the Hamming distance that measures the minimum number of substitutions required to change one string into the other as a selection criterion, one chromosome is selected from every randomly selected pair of chromosomes on a competitive selection process. The selection process ensures that the diversity of chromosomes is reserved during the selection process.

2.2.4 Crossover operator: Crossover is an exchange of genetic material between homologous individuals for final genetic recombination. While many crossover operators available in genetic algorithm, this research employs the adaptive genetic operator proposed by Srinvias and Patnaik (1994). The crossover probability P_c is used to allow the crossover between chromosomes. This probability value changes continuously with the change of fitness value during the search process. This crossover probability is defined as:

$$P_c = \begin{cases} P_{c1} - \dfrac{(P_{c1} - P_{c2})(f' - f_{avg})}{f_{max} - f_{avg}} & f' \geq f_{avg} \\ P_{c1} & f' < f_{avg} \end{cases} \tag{6}$$

where

P_{c1} is the maximum crossover probability, the value is 0.95;

P_{c2} is the minimum crossover probability, the value is 0.45;

f' is the fitness value;

f_{max} is the maximum fitness value; and

f_{avg} is the minimum fitness value.

Mutation operator: In genetic algorithms, mutation is used to maintain genetic diversity from one generation of a population of chromosomes to the next. Similar to the crossover operators, the mutation operator proposed by Srinivas and Patnaik (1994) was adopted in this research. The mutation probability P_m, is defined as:

$$P_m = \begin{cases} P_{m1} - \dfrac{(P_{m1} - P_{m2})(f_{max} - f_{avg})}{f_{max} - f_{avg}} & f \geq f_{avg} \\ P_{m1} & f < f_{avg} \end{cases} \tag{7}$$

where

P_{m1} = maximum mutation probability, 0.1

P_{m2} = minimum mutation probability, 0.001

f_{max} = maximum fitness value

f_{avg} = minimum fitness value

f = fitness value

2.2.5 Threshold of conversion probability: The AGA technique was used to minimize the fitness function $f(x)$ corresponding to the selected spatial samples. With the minimum value of $f(x)$, a set of optimized chromosome can be achieved together with its defining parameters. This leads to the generation of the conversion probability of each cell from non-urban to urban in the urban growth process. Hence, by comparing the conversion probability of a cell at time t with a pre-defined threshold value (Wu 1998; Wu 2002; Li and Yeh 2002b), if the conversion probability of the cell at time t is larger than the pre-defined threshold value the cell will be converted to an urban state at the following step. Otherwise, the state of the cell will remain unchanged.

$$S_{ij}^{t+1} = \begin{cases} Urban, & if\ P_G^t > P_{\text{threshold}} \\ Non\text{-}urban, & if\ P_G^t \leq P_{\text{threshold}} \end{cases} \tag{8}$$

3. APPLICATION AND RESULTS

3.1 Study area and data

The proposed AGA-CA model was applied to simulate the urban growth in Shanghai's Jiading District, which is located in the Yangtze River Delta of Eastern China. The study region consists of eleven blocks (towns) with a total area of 463.6 km^2. Rapid urban expansion had occurred in the 1990s due to the fast economic development and population growth. Urban growth of this region from 1989 to 2006 was mapped out using data from various sources, including two Landsat-5 Thematic Mapper (TM) images acquired on August 6, 1989 and April 30, 2006 respectively to obtain spectrum information of the study area. In addition, essential ancillary data include a 1:50,000 topographic map, cadastre and transportation maps which were collected from the local government.

3.2 Model configuration and implementation

While many spatial factors can make an impact on urban development, in practice, not all factors can be quantified into a simulation model, especially when data reflecting such factors are either not available or not accessible. Considering the process of Jiading urban growth historically, urban development in this district is largely related to the distribution of existing urban and towns, accessibility to transport as well as preservation of primary agriculture land. Therefore, five spatial factors were selected, including distance to urban centre (x_1), distance to town centre (x_2), distance to main roads (x_3), distance to cropland (x_4), and distance to orchard field (x_5). The impact of each factor on urban development may be different, hence, different weights were assigned to each of this factors which were represented by a_1, a_2, a_3, a_4, a_5, respectively.

A total of 5000 sample cells were randomly selected from the Landsat TM images for the AGA model to commence the searching process. The distances of each of these samples to the urban centre, town centre, main road, cropland and orchard field were computed in GIS. These distance values were normalized to have a standard value ranging from 0 to 1.

A modelling framework was developed within ESRI's ArcGIS environment based on Microsoft Visual Basic .NET and ArcGIS Engine 9.2 technologies. This modelling framework incorporates the AGA-CA model as well as a number of other CA based modelling approaches. The user-friendly graphical user interface makes ease the sophisticated computation process of the model (Figure 1).

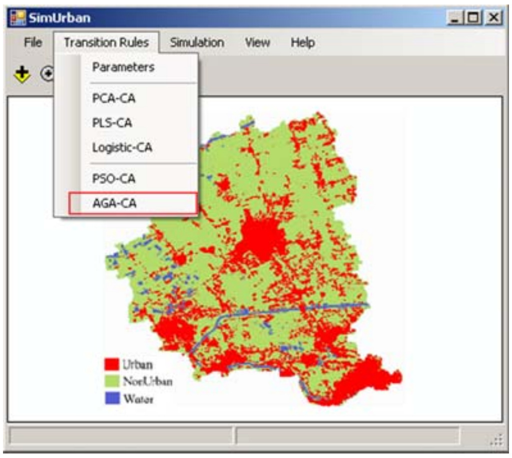

Figure 1. GUI of the modelling framework

Moreover, the strong coupling of the model within a GIS framework makes it possible to use the various display and analysis functions of GIS in raster based data integration and modelling. Hence, this modelling framework becomes an important component of the AGA-CA program for modelling urban growth.

3.3 Results and Discussion

Using the adaptive genetic algorithm proposed in this research, the model was executed to start the search and optimization process with the sample data selected from the 1989 Landsat TM imagery. Figure 2 shows the fitness track in the evolutionary computation of the AGA model, which demonstrates a rapid convergence rate after over 30,000 times of iteration, with a convergence fitness value of 391.9855 (Figure 2).

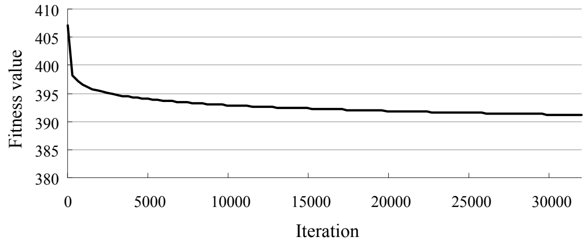

Figure 2. Fitness track of the AGA model

The convergence of the fitness track leads to the identification of a set of optimized chromosome or solution as:

$$C = \begin{cases} a_1 = -0.5083 \\ a_2 = -0.6417 \\ a_3 = -0.4962 \\ a_4 = +0.3723 \\ a_5 = +0.2174 \end{cases} \tag{9}$$

In fact, this optimized solution becomes the initial input data for the CA model to compute the conversion probability of cells from non-urban to urban state. Hence, Equation (3) can be re-written as:

$$\begin{aligned} P_G &= [1+\exp(-(-0.5083x_1 - 0.6417x_2 \\ &- 0.4962x_3 + 0.3723x_4 + 0.2174x_5))]^{-1} \\ &\times P_\Omega \times con(cell_{ij}^t = suitable) \times (1+(-\ln\gamma)^\beta) \end{aligned} \tag{10}$$

The optimized chromosome displayed in Equation 9 shows different effect of the spatial factors on urban growth in Shanghai's Jiading District. According to Equation 9, a negative $a_i (i = 1,...,5)$ will lead to a larger P_G value, i.e., a higher possibility for a cell to convert from non-urban to urban state. Likewise, a positive a_i will result in a lower P_G value, hence, a lower possibility for the cell to be converted into an urban state in the next time step. The optimised result generated from the AGA approach shows that the distance to town centres has the most significant impact on the development of cells within its neighbourhood. This is reflected by the smallest value of a_2 ($a_2 = -0.6417$). Likewise, the spatial proximity of a cell to urban centre and main road also positive roles to its urban development (with $a_1 = -0.5083$ and $a_3 = -0.4962$ respectively). On the other hand, factors such as distances to cropland and orchard field have negative impact on urban development (with $a_4 = 0.3723$ and $a_5 = 0.2174$). Hence, the close a cell is to cropland and orchard field, the less opportunity the cell is to be developed into an urban state. This is largely in consistent with the conservation of primary agricultural land policies in practice.

Using the urban distribution pattern defined from the 1989 satellite image classification as the initial input data for the urban CA model (Figure 3a), and the transition rules generated from the AGA approach, the CA model was operated to generate a series of urban scenarios. Each iteration of the model represents one year. After 16 iterations the model generates a map representing urban growth patterns of Jiading District in 2006 (Figure 3c). This simulated urban scenario was compared with the actual urban distribution as defined by classifying the 2006 Landsat TM image (Figure 3b).

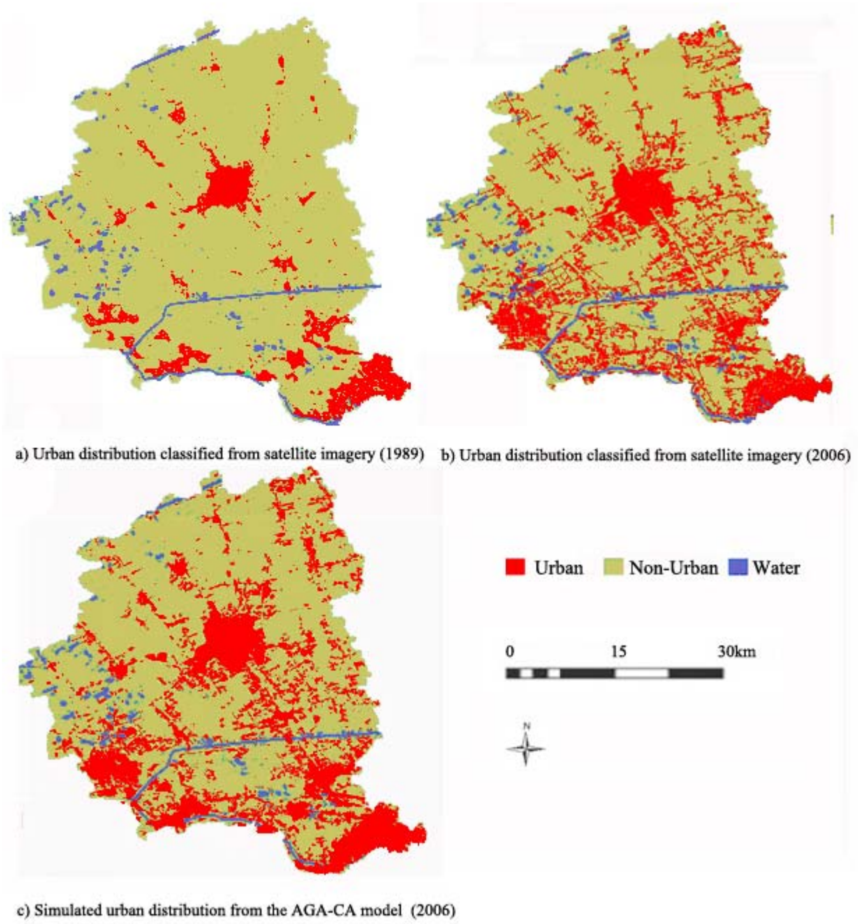

Figure 3. Actual and simulated urban scenarios of Jiading District using the AGA-CA model

Figure 3 b and c show high similarity even by visual inspection and comparison. By comparing the two maps on a cell-by-cell basis, an error matrix analysis was carried out (Table 1). The results show that the producer's accuracy for non-urban and urban areas were 85.7 and 76.3 per cent respectively, while the user's accuracy for the non-urban and urban categories were 81.1 and 86.7 per cent, respectively. Consequently, the model generated an overall accuracy of 82.7 per cent and a Kappa coefficient of 60.9 per cent. These simulation accuracies are considered good given that only five spatial distance variables were considered in the model. Should other factors such as the social demographic controls, institutional policy effects concerning sustainable urban development as well as other economic constraints included into the model, the AGA-CA model would also be able to can be used to generate and evaluate various urban growth scenarios.

		Simulation Results		
		Non-urban	Urban	Row Total
Satellite-based Land Use Classification	Non-urban	31556	5261	36817
	Urban	4115	13211	17326
	Column Total	38901	15242	54143
		Producer's Accuracy		Omission Error
Non-urban		85.7%		14.3%
Urban		76.3%		23.8%
		User's Accuracy		Commission Error
Non-urban		81.1%		18.9%
Urban		86.7%		13.3%
Overall Accuracy			82.7%	
Kappa Coefficient			60.9%	

Table.1 The confusion matrix between remote sensing-based land use classification and the simulated urban categories using the AGA-CA model of Jiading District in 2006

4. CONCLUSION

This paper presents an improved CA model optimized by adaptive genetic algorithm technique, which has been widely used as an evolutionary computation technique. By using the adaptive genetic algorithm technique, a set of transition rules and their defining parameters have been identified and optimised using the limited data available as input data sources. The AGA technique is particularly useful in optimizing the CA transition rules which can be used by conventional CA models based on logistic regression approach. The application of the APA-CA model in Shanghai's Jiading District demonstrates the effectiveness of the AGA technique in transition rule optimization for CA based urban models, which can contribute positively to human studies on urban dynamics.

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