

## LINEAR FEATURE ALIGNMENT BASED ON VECTOR POTENTIAL FIELD

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### ABSTRACT:

An approach to align a linear feature in one dataset with a corresponding feature in another dataset that is considered more accurate is presented. The approach is based on the active contours (snake) concept, but implements the external force as a vector potential field in which case the source of the force is in vector form; further the snake feature is implemented as a non-closed snake. This is different from the conventional implementation of the snake, where the source of the external force is an image and the force is implemented as a gradient flow and usually as a closed snake.

In this approach two conditions: the length and alignment conditions have to be satisfied to obtain a good alignment. Whereas the length condition ensures that the length of the snake feature is nearly equal that of the reference feature, the alignment condition requires that the snake and the reference feature are properly aligned. The length condition is achieved by fixing the end points of the snake feature to those of the reference feature. The alignment condition is achieved by segmenting the reference feature so that there is uniform external force from all parts of the feature. One assumption in this approach is that the snake and the reference feature are matched prior to alignment. An outstanding challenge therefore is to find out how to consider the effects of non-corresponding but neighbouring reference features on a snake feature in circumstances where prior matching has not been undertaken.

### 1. INTRODUCTION

Feature alignment is one of the most critical steps in geometric data integration. It entails a geometric adaptation of features in two different datasets to obtain a better geometric correspondence between them, in which one of them is considered to be of a better geometric accuracy. Feature alignment is generally achieved through geometric transformations and depending on the application, different techniques can be used in feature alignment.

In a data integration problem, feature alignment is usually carried out prior to, or after feature matching. In the former case, feature alignment would involve a global transformation, while the latter would involve a local non-rigid transformation in addition to a global transformation. When the scale (resolution) and the accuracy of the datasets involved is almost similar, the expected discrepancies in position after a global transformation are not so large, otherwise the discrepancies could be quite significant.

The most common method used for feature alignment is the Iterative Closest Point (ICP) algorithm introduced by Besl and McKay (1992). ICP is a rigid transformation based on the 4 parameter Helmert transformation for a 2D case, although any transformation can be used and requires a good initial alignment. ICP takes the closest point, the problem however is that ICP will eventually match a point that is closest among all the candidate points even though the distance is very far. In this case, ICP can be implemented by specifying a certain threshold distance. An approach that does not require a specification of the threshold distance would be required in particular where the relative accuracies of the datasets involved in the alignment is not known. Active contours or snakes as commonly known have been used for feature alignment without the need for a threshold distance. Although other parameters such as the segmentation distance of reference feature, which affects the final alignment, have to be specified.

An active contour or snake is a force minimizing curve that is influenced by internal force coming from the curve itself and external forces computed from the image data (Kass et al., 1988). A curve is referred to as an active contour by the fact that the curve segments are adjusted iteratively by moving the control points that connect the vertices to lock on the edges of an image. Snakes may be understood as a special case of a more general technique of matching a deformable model to an image by means of energy minimization.

A number of modifications to the original concept of the active contours have been suggested and implemented. In most of the modifications and implementations however, the snakes form closed loops in the image, although this is not necessarily true for all the snakes. Based on the original concept, most implementations convert the reference feature to an image even when it is in vector format, in which case the external force is then modelled as a gradient flow.

In this paper, an approach that implements a non-closed snake and the external force as a vector field is presented. The development of this approach was motivated by a problem in data integration, in particular feature alignment of two vector datasets, i.e., the snake and the reference feature are both in vector format. Furthermore, the accuracy of the snake feature is unknown (Siriba, 2009). The approach wants to avoid converting the reference feature to an image.

The next section of this paper presents an overview of active contours. This is followed by a discussion of vector based snakes that form the basis of the approach presented in this paper. The experimental results of the implementations of the individual and combined forces is presented, which is then followed by the evaluation of the quality of the alignment using relative curve length and the Hausdorff distance. Conclusions and suggestions for further work come at the end of the paper.

## 2. RELATED WORK

The original definition of active contours or snakes is that they are curves defined within an image domain that can move under the influence of internal forces coming from within the curve itself and external forces computed from the image data. In most image processing applications, snakes are used particularly to locate object boundaries, model shapes, segmentation of images, motion tracking (Xu and Prince, 1998). In another novel application, snakes have been used in the smoothing of line objects within the framework of linear feature generalization (Burghardt, 2005) and for displacement (Burghardt and Meier, 1997).

The snake as a spline is represented as a parametric curve as:

$$V(s) = (x(s), y(s)) \quad (1)$$

Where (s) is proportional to the curve length, x and y are the curve coordinates.

The snakes's force function is composed of internal and external force components and is given as:

$$E_{snake} = \int_0^1 E_{snake}(V(s))ds = \int_0^1 (E_{internal}(V(s)) + E_{external}(V(s)))ds \quad (2)$$

In discrete form (2) could be represented as:

$$E_{snake} = \sum_{i=1}^n (E_{internal}(i) + E_{external}(i)) \quad (3)$$

Where  $i$  denotes a vertex point on the snake and  $n$  is the total number of vertices in the snake.

The internal spline force is composed of the first and the second order terms, which are the first and second derivatives of the curve. The first (elasticity) term keeps the snake from stretching or contracting along its length, while the second (bending) term keeps the snake from bending too much. The internal force can be represented as:

$$E_{internal} = \alpha(s)|v_s(s)|^2 + \beta(s)|v_{ss}(s)|^2 / 2 \quad (4)$$

Where  $v_s$  and  $v_{ss}$  are the first and the second derivatives,  $\alpha(s)$  and  $\beta(s)$  are functions of the arc length along the snake. These are similar to  $R$  and  $K$  variables in (6). In discrete form (4) becomes:

$$E_{Internal}(i) = \alpha_i |v_i - v_{i-1}|^2 + \beta_i |v_{i-1} - 2v_i + v_{i+1}|^2 / 2 \quad (5)$$

The external force provided by the image is a weighted combination of force functional which attracts the snake to lines, edges and terminations. These energies are functions of

the image intensity, image gradients and curvature of level line respectively. A discussion on the implementation and numerical methods are available in (Kass et al., 1988).

In the various implementations of the snakes, there are notable differences in some basic aspects. For instance, the snakes form closed loops in the image, although not necessarily true for all the snakes. This means that open snakes are also possible as implemented in (Burghardt and Meier, 1997; Burghardt, 2005). Moreover, for closed snakes, depending on the problem and the initialization, the snake can be made to grow or shrink, although the latter case is common.

The active contours as initially conceived require edges in an image onto which they are accurately localized (Kass et al., 1988). The edges provide the external force to pull the snake. Although the external force is modeled based on the image intensity or the magnitude of the image gradient, the external force can be modeled as a gravitational field (Honea et al., 2002), in which case the snake is attracted to edges in the image even if they are some distance away.

Active contours are further differentiated as either parametric or geometric. While the original model proposed by (Kass et al., 1988) constitutes a parametric implementation, a geometric implementation is proposed by (Caselles et al., 1993). In both models, most implementations define single closed object boundaries. In a situation where multiple objects have to be handled, the maintenance of the topology is important. A method called network snakes that incorporates a complete topological and shape control was introduced by Butenuth (2008).

Another aspect of active contours which differ in concept and implementation is the manner in which the force is minimized. Some of the methods used to minimize the force of the active contour include finite differences as used in the Eulerian equations (Kass et al., 1988), dynamic programming and greedy algorithm (Lam and Yan, 1994).

These differences in the various aspects of active contours provide an opportunity for various combinations for different applications. While the approach presented in this paper generally uses the original concept, it differs with most implementations in that the source for the external force does not have to be converted to an image as used for instance in (Song et al., 2006, Burghardt, 2005) for feature displacement and line smoothing respectively. As a consequence of this, the external force is modelled as a vector potential field instead of image gradient. Although this approach has considered the snake paradigm proposed by Honea et al. (2002), it differs from the concept by considering the source of the external force as a vector instead of an image. Although Bader (2001) modelled active contours as force vectors rather than as an image, it is informed by the traditional concept of snakes. Again, in this implementation, the snake is assumed to be an open snake as opposed to closed snakes which are very common, particularly in image processing. In general this approach uses geometric implementation that uses finite elements method to minimize the forces. The next section presents the theoretical concepts upon which this approach is based.

## 3. SNAKES BASED ON VECTOR POTENTIAL FIELD

The problem in feature alignment is that two linear features deemed to be similar but are not in perfect correspondence after initial alignment are required to be brought to a perfect

correspondence. One of the features is considered fixed (the reference) while the other one, in this case referred to as the snake is iteratively moved until it is aligned with the reference feature.

To implement the alignment within the vector domain, reference is made to the paradigm proposed by Honea (2002), in which the total force ( $\bar{F}$ ) acting on a point (vertex) on the snake is given as:

$$\bar{F}_i = w_e \bar{E}_i + w_1 \bar{R}_i + w_2 d_k \bar{K}_i \quad (6)$$

Where  $w$  are the weights,  $\bar{E}_i$  is the external force, while  $\bar{R}_i$  and  $\bar{K}_i$  are the elastic and the bending components of the internal force respectively acting on a point on the snake.

### 3.1 External Force

Let  $X = \{\bar{x}_i\}, i=1,2...m$  denote the vertices of the snake and  $S = \{\bar{s}_i\}, i=1,2...m$  denote the vertices of the reference linear feature as illustrated in figure 1.

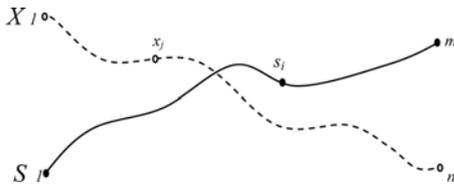


Figure 1: Snake and reference linear features

The net external force from the vertices of the reference feature on apoint (vertex) on the snake is then represented as:

$$\bar{E}_i = \sum_{j=1}^m E_{ij} \frac{\bar{s}_i - \bar{x}_j}{|\bar{s}_i - \bar{x}_j|} \quad (7)$$

In which  $E_{ij}$  is the gravitational field potential of the vertex,  $s_i$  in the reference feature on the snake vertex,  $x_j$  and is given as:

$$\bar{E}_{ij} = \alpha \frac{e(\bar{s}_j)}{d_{ij}^2} \quad (8)$$

Where  $d_{ij}$  is the Euclidean (or other metric) distance from  $\bar{x}_i$  to  $\bar{s}_j$ ;  $e$  and  $\alpha$  are constants and are taken to be unit.

### 3.2 Internal Force – Elastic Component

The internal force of the snake is composed of two parts like in the traditional case. If  $C_i$  denotes the snake point at  $x_i$  and let  $C_{ix}$  and  $C_{iy}$  be the x and y-coordinates of  $C_i$ , then the elastic force acting on a snake point from all other snake points is expressed as:

$$\bar{R}_i = \left( \sum_{j \neq i} \frac{1}{C_{ix} - C_{jx}}, \frac{1}{C_{iy} - C_{jy}} \right) \quad (9)$$

This force is responsible for maintaining the topology of the snake feature.

### 3.3 Internal Force – Bending Component

Let  $C_{i-1}$ ,  $C_i$  and  $C_{i+1}$  be three consecutive snake points as illustrated in figure 2 and  $K$  be the unit vector normal to  $(C_{i-1}, C_{i+1})$ , with its magnitude proportional to the distance  $d_k$ .

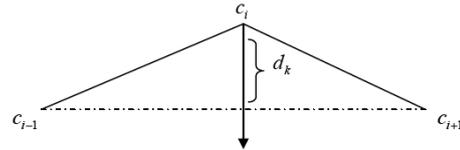


Figure 2: Internal Force (Bending Component)

### 3.4 Combining the Forces

Equation (6) shows the net force acting on a snake point. While the main issue in the equation is how to determine the weights, two conditions must however be fulfilled:

- i) The length of the final snake should be the same or nearly the same as that of the reference snake (length condition)
- ii) A perfect or near perfect alignment of the snake with the reference feature should be achieved (alignment condition).

Therefore the forces can be combined using any weights as long as these conditions are achieved. The next section presents an analysis of the effects of these forces based on an experimental snake and reference feature.

## 4 EXPERIMENTAL RESULTS

### 4.1 External Force

The external force is considered the most important force in the system because it is the one that will eventually influence the snake to move to the required position. Ideally, the snake should eventually be aligned with the feature that provides the external force. However, depending on the number of iterations (dynamism) and conditions specified for the system, the snake could behave in a number of ways, for instance, if the number of iterations is too small, the snake may not move to its final alignment with the reference feature, otherwise it will move beyond the required alignment position.

A linear feature consists of vertices that are normally not uniformly spaced, in which case the feature is considered to be unsegmented. If additional vertices are introduced at equal intervals the feature could be considered to be segmented. This operation is necessary in the implementation of the external force to ensure uniform pulling force from the reference feature. This is similar to setting the resolution of the image in the conventional snake as small as possible to ensure uniform external force.

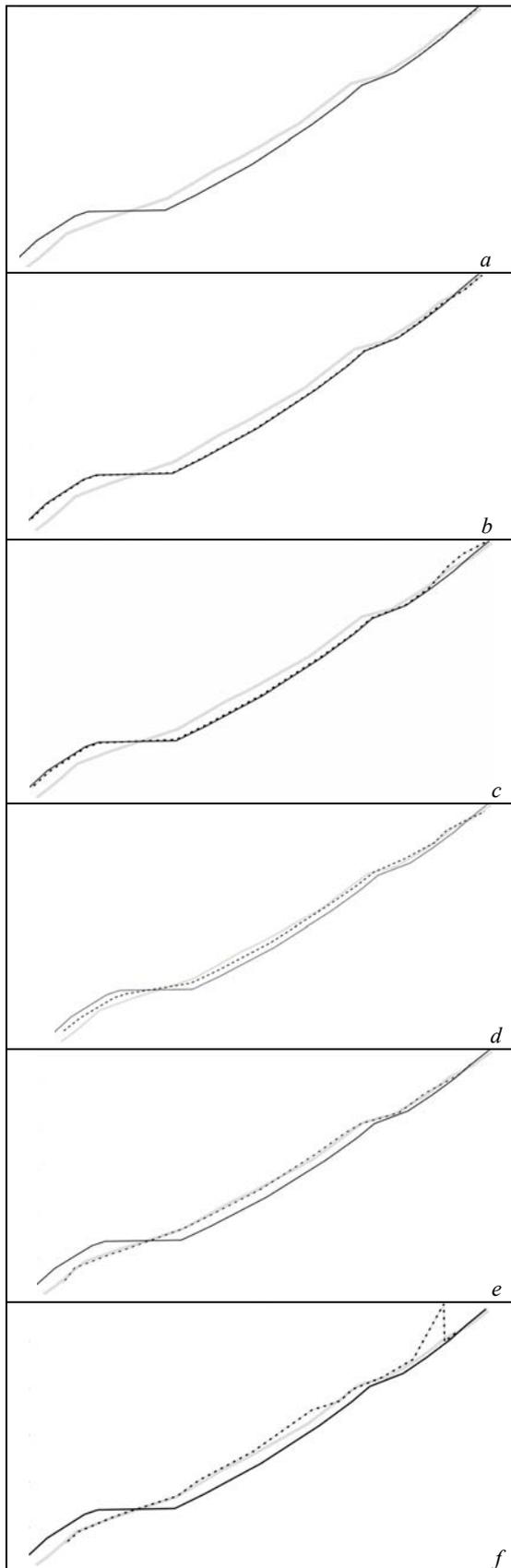


Figure 3: External Force after 2, 5 and 10 iterations for the unsegmented (a,b,c) and segmented (d,e,f) reference feature respectively

Figure 3 illustrates the aligned position of the snake (in dashed black line), the reference feature (bold gray line) and the initial snake position (black line) after applying only the external force. In the figure, a, b and c shows the result after applying the external force at 2, 5 and 10 iterations for unsegmented reference feature, while d, e and f are the results for a segmented reference feature for the same number of iterations. The length of the reference feature is approximately 470m and the segmentation distance is 1.0m.

It is noted that, for the unsegmented reference feature a lot more iterations are required to bring the features into alignment, however for segmented reference feature, alignment is achieved quite fast. The downside of segmenting the reference feature is that the snake shrinks as the number of iterations increases and unwanted kinks are introduced after certain number of iterations, as a result of possible infinitesimal distance between points on the snake and the reference feature.

The length of the example reference feature is 471.161m, while the original length of the snake feature is 478.528m. Generally, the snake feature shrinks with successive iterations. This is because the snake will tend to be pulled towards the centroid of the reference feature. The rate of shrinkage will depend on the relative forces of the snake and the corresponding reference feature. In particular, the force from the corresponding reference feature depends on the number of points (vertices) on the reference feature.

#### 4.2 Internal Force – Elastic and Bending Components

Figure 4 illustrates the elasticity effect on the example snake, with bold gray line representing the initial snake position and the dotted black line representing the final snake position after 10 iterations. As the number of iterations increases, the snake reduces to a stretched and smoother line, which is almost a straight line. In other words, some vertices in the feature can be eliminated without compromising the structure of the snake feature.

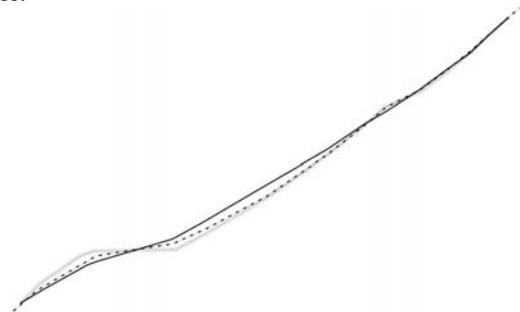


Figure 4: Initial snake position (in bold gray), final position after applying the elastic force (black line) and the bending force (dotted black line)

The bending force minimizes the curvature of the snake by pulling a vertex towards the line between its two neighbouring vertices. This force ensures that the successive structure of the snake points is maintained even when the external force may tend to distort that structure. If implemented separately, this force will result in a snake which has maintained its length since the end points do not experience any bending. In figure 4, the effect of the bending internal force on the example snake after 10 iterations is illustrated, with the dotted black line. The net effect of the bending force is the retaining of the initial character of the snake, and the effect of the force is equivalent to line simplification.

Since this force does not consider the reference feature and the snake points change their positions, the net effect is similar to a simplification of a linear feature. The extent of simplification depends on the number of iterations specified during execution. Overall, the internal forces ensure that the topologic structure of the snake is maintained.

#### 4.3 Combined Forces

During the implementation, the best approach is to combine the individual force components and then compute the snake displacements iteratively. This is as opposed to iteratively computing the individual components separately and then combining them.

Combining the forces results in the net force acting on a snake point, this could be acceleration or displacements with instantaneous velocity depending on the nature of dynamism defined in the system. Dynamism is defined through the parameters – i.e., the weights and the number of iterations. Figure 5 (a) illustrates the snake position (in dotted black line) after 10 iterations, the reference feature (in bold gray) and the initial snake (in black) after combining the forces with equal weights.

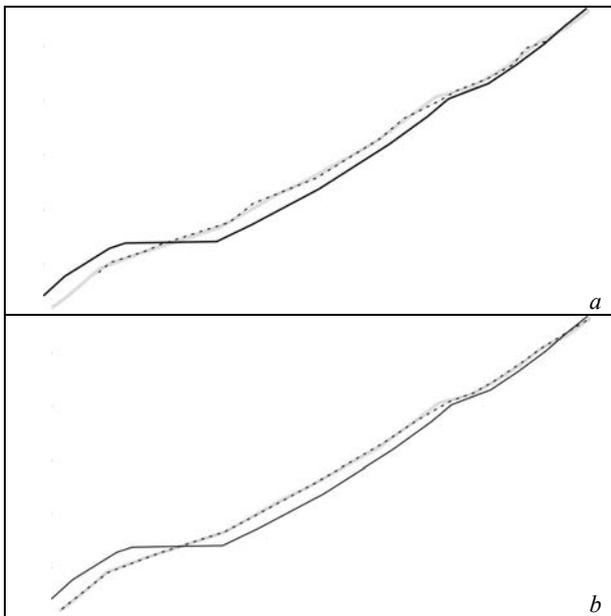


Figure 5: Aligned snake feature without conditions (a) and with conditions (b)

It can be observed in figure 5 (a) that even after combining the forces, the final snake has contracted. This is because equal weights were assumed for the forces. So a strategy to determine the weights is required so that the two conditions stated in section 3.3 are simultaneously achieved.

##### (i) Length condition

During the application of the forces, the elastic and bending force ensures that the length of the snake feature does not change significantly, while the external energy is one that affects the length of the snake as it evolves. Therefore the length condition can only be fulfilled by considering mainly the external energy. If left without including some constraint, the

final length of the snake feature under the influence of the reference feature will increase or decrease, for example as illustrated in figure 5 (a). This is because the end points of the snake feature are not only affected by the terminal points of the reference feature but also by the intermediate points (vertices) of the reference feature. The effect of the intermediate points on the terminal points of the snake can be cancelled by fixing the coordinates of the terminal points of the snakes to the terminal points of the reference feature.

##### (ii) Alignment condition

Once the terminal points are fixed, the alignment condition has to be fulfilled, which requires the vertices of the snake to be aligned with the reference feature. In this case the relative importance of the forces should be considered. Since the ultimate objective is to achieve a near perfect alignment, the external force carries more weight compared to the elastic and bending forces. The elastic force is partly considered in the length condition, however, the intermediate snake vertices need to be spaced appropriately with regard to the initial snake configuration. In this case a way to determine the weights has to be established as described in the next paragraph.

##### (iii) Weights

To fulfill the alignment condition, an appropriate strategy for determining the relative weights is required. Some of the strategies that can be considered include the following: (i) assigning equal weights or (ii) assigning relative weights.

Assignment of equal weights for the forces has been adopted in this implementation and figure 5 (b) illustrates the finally aligned snake.

One of the implied conditions is to maintain the structure of the snake as much as possible. This condition therefore does not allow for the segmentation of the snake, because additional vertices would be introduced. Since no additional vertices would be introduced, there won't be a perfect alignment of the snake with the reference feature; at least some sections will be misaligned.

#### 4.4 Evaluation of the Quality of the Alignment

The quality of the alignment can be evaluated in terms of the ratios of the curve lengths and the maximum displacements between the snake and the reference feature. For positional misalignment, the classical Hausdorff distance (Hangouet, 1995) can be used as a quality measure. Whereas the ratio of the curve lengths is a good parameter for quality evaluation, the Hausdorff distance is a better indicator for the alignment quality because it indicates by how much the positional error of the snake feature has been improved, in case the objective of the alignment was to enhance the positional accuracy.

Table 1 shows for the values for the curve length ratios and the Hausdorff distances after the alignment for various cases. A value of 1.0 for the ratio of the curve lengths of the snake and reference feature would indicate a perfect alignment. Unless there is a perfect alignment, this value would usually vary and the value of the variation from 1.0 is proportional to the quality of the alignment. The initial snake to reference feature length

ratio and the Hausdorff distance were 1.0156 and 13.77m respectively.

Calculating the Hausdorff distance for the final case when the conditions are fulfilled is straight forward; however for the other cases this is not trivial. This is because the length of the snake feature changes significantly, further, the distance evaluated for the unsegmented cases may be misleading, because the number of vertices in the reference feature are few and not uniformly distributed.

Case	Snake to Reference Length Ratio	Hausdorff Distance (m)
Figure 3 (a)	1.0136	19.46
Figure 3 (b)	1.0125	19.35
Figure 3 (c)	1.0119	19.17
Figure 3 (d)	0.9641	7.56
Figure 3 (e)	0.9132	2.75
Figure 3 (f)	1.0021	30.91
Figure 4 – Elastic force	1.0197	13.21
Figure 4 – Bending force	0.9921	12.04
Figure 5 (a)	0.8210	7.34
Figure 5 (b)	0.9955	1.28

Table 1: Evaluation of Feature Alignment

In the experiment the accuracy of the reference feature is considered to be higher than that of the snake feature. A better evaluation of the quality of the alignment would be achieved if the relative accuracies of the features is known before the alignment process.

## 5 CONCLUSIONS AND FURTHER WORK

A segmented reference feature provides a fast alignment, but after a certain number of iterations, the snake starts to shrink and even unwanted kinks are introduced. This is overcome by fulfilling the length and alignment conditions as described here. Adaptive determination of the number of iterations is part of further being considered.

In general, the approach presented here could be useful for feature alignment applications that do not necessarily require the snake features to be closed and where the accuracy of the snake feature is so low that using buffers as the basis for alignment may not be sufficient. From the example illustrated here the positional accuracy of the snake feature was improved from 13.77m to 1.28m. Although this is a general indicator for the quality of the alignment, considerations for the relative accuracies of the features will constitute a better quality assessment. A further investigation will involve establishing whether the distance used to segment the reference feature affects the overall result, particularly the Hausdorff distance.

Although the assignment of equal weights to the forces seemed sufficient, it would be interesting to establish a way to determine the relative weights for the forces as possible further work.

The approach discussed herein was based on the assumption that the features to be aligned are already matched to their corresponding reference features. This assumption keeps the snake feature from the effects of the external forces of other reference features. Further work will also investigate, if different matching candidates can be evaluated in an integrated

way using their attractive forces. In this way, reference features which are closer will have a higher influence than features that are further away. This would allow to start the adaptation without prior matching.

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