The story of bundle adjustment: Karsten Jacobsen’s chapter

Stewart Walker
photogrammetry4u llc, San Diego, California, USA

Christian Heipke
Institute of Photogrammetry and GeoInformation (IPI), Leibniz Universität Hannover, Germany

Words of thanks

Stewart Walker is grateful to have received funding from

- Deutsches Gesellschaft für Photogrammetrie und Fernerkundung (DGPF)
- The Keynote Speakers program of the International Society for Photogrammetry and Remote Sensing (ISPRS)

DGPF

isprs

information from imagery

Motivation

- Provide an interesting keynote about the technology at the very heart of photogrammetry
- Acknowledge the enormous contribution of Karsten Jacobsen
  - Development of bundle adjustment
  - Application of bundle adjustment
- Put this in the context of almost 100 years of development
- Attempt to discern current directions

My life and Karsten's

Dr.-Ing. Karsten Jacobsen
Institut für Photogrammetrie
und Ingenieurvermessungen
Universität Hannover

30167 Hannover, den 14.10.199
Nienburger Straße 1
Geschäftszimmer 0511-762 2482
Sachbearbeiter 0511-762 2485
Telefax 0511-762 2483

- 2 -

3. The installation is more or less only a copy of the software + some
minor arrangements (like path definition).
The training at the customer site can be done by us, here we do have

HANNOVER

Dr.-Ing. Karsten Jacobsen
Institut für Photogrammetrie
und Ingenieurvermessungen
Universität Hannover

30167 Hannover, den 14.10.1993
Nienburger Straße 1
Geschäftszimmer 0511-762 2482
Sachbearbeiter 0511-762 2485
Telefax 0511-762 2483

Leica AG
z.Hd. Herrn Dr. A. Steward Walker
CH-5035 Unterentfelden

FAX 0041-64-43 07 34

Program System BLUH

4 years: 20% of licence fee,
6 years: 30% of licence fee

- 2 -

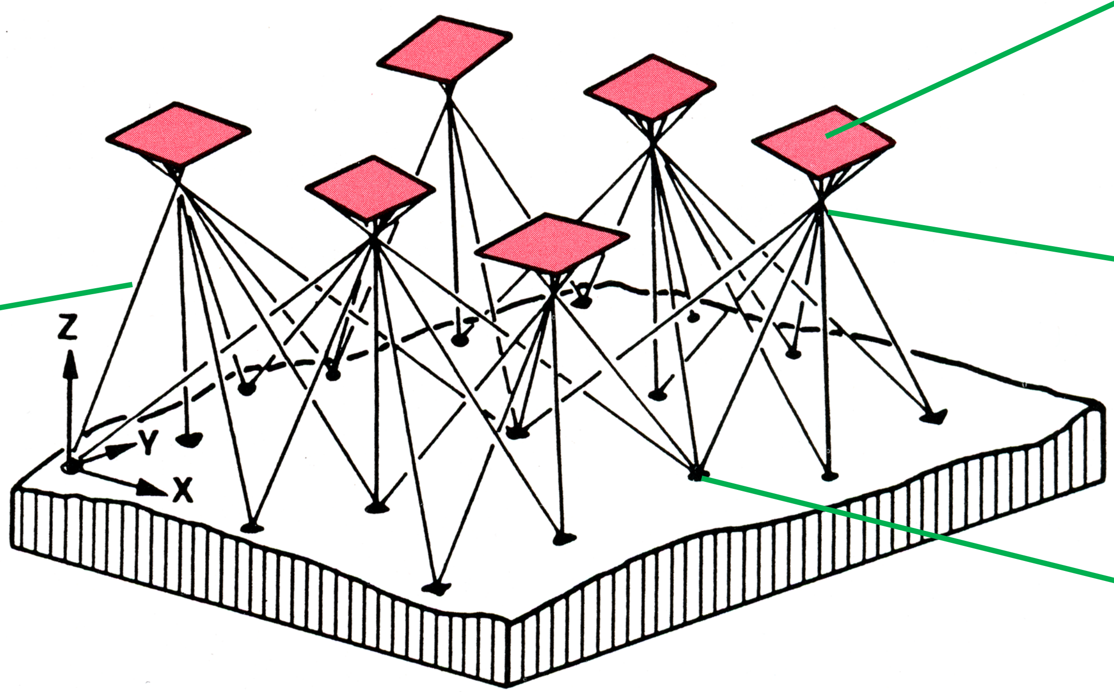
Topics

- What is bundle adjustment?
- Bundle adjustment in the analog era
- Bundle adjustment in the analytical era
- Bundle adjustment in the digital era
- Classical versus computer vision approaches
- Bundle adjustment today
- Current directions

What is bundle adjustment*?

* Many authors use the term ”bundle block adjustment” or “BBA”

Bundles of rays modeled by collinearity equations or projective geometry formulation



coordinates of points measured on image

camera parameters: position and attitude; calibration

coordinates of ground point, of which some can be control or check points

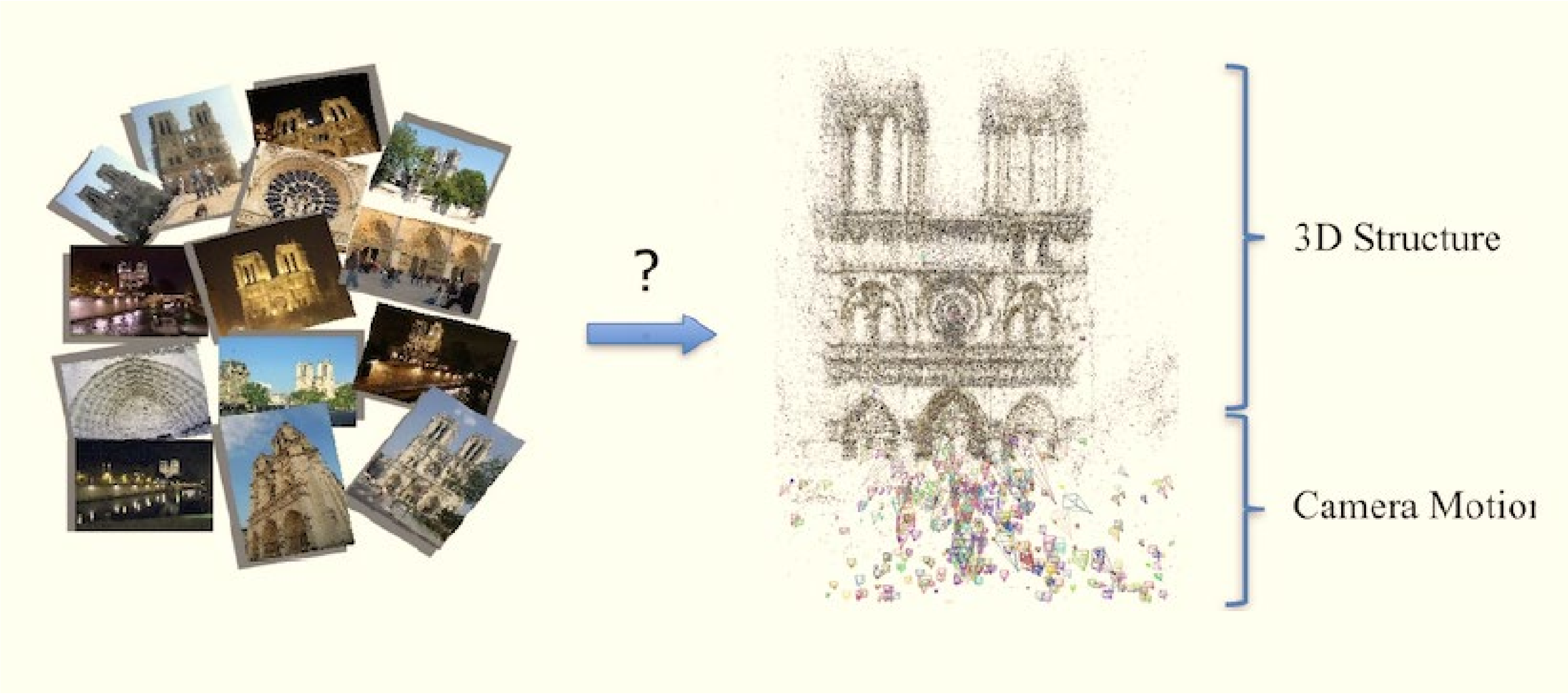
“Bundle adjustment (BA) is an optimization process refining the estimates of extrinsic camera parameters (position and orientation, or pose) and the three-dimensional (3D) positions of features using overlapping images from multiple views..” (Moore *et al.*, 2009, 1)

Another definition of bundle adjustment

"Bundle adjustment is the problem of refining a visual reconstruction to produce jointly optimal 3D structure and viewing parameter (camera pose and/or calibration) estimates. Optimal means that the parameter estimates are found by minimizing some cost function* that quantifies the model fitting error, and jointly that the solution is simultaneously optimal with respect to both structure and camera variations. The name refers to the 'bundles' of light rays leaving each 3D feature and converging on each camera centre, which are 'adjusted' optimally with respect to both feature and camera positions. Equivalently — unlike independent model methods, which merge partial reconstructions without updating their internal structure — all of the structure and camera parameters are adjusted together 'in one bundle'." (Triggs et al., 1999, 298-299)

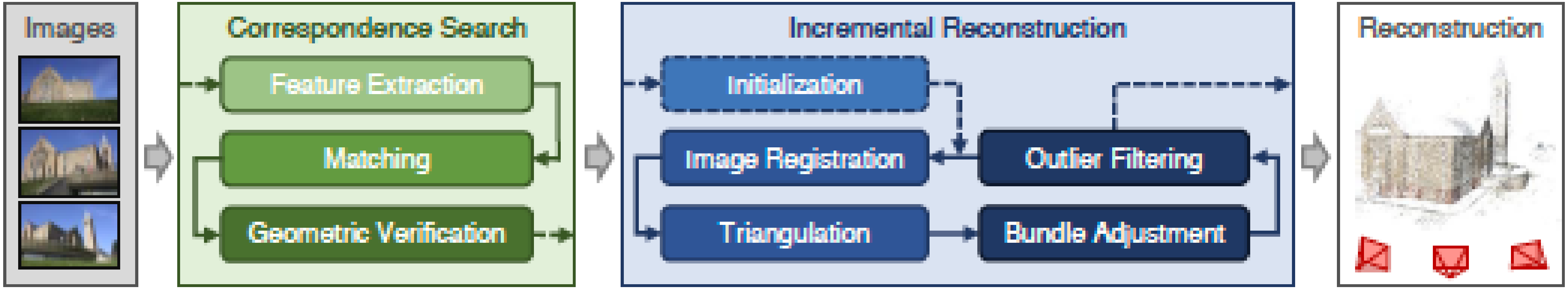
* "… optimizing a complicated nonlinear cost function (the total prediction error) over a large nonlinear parameter space (the scene and camera parameters)." (*ibid*., 302)

Computer vision sketches of the problem

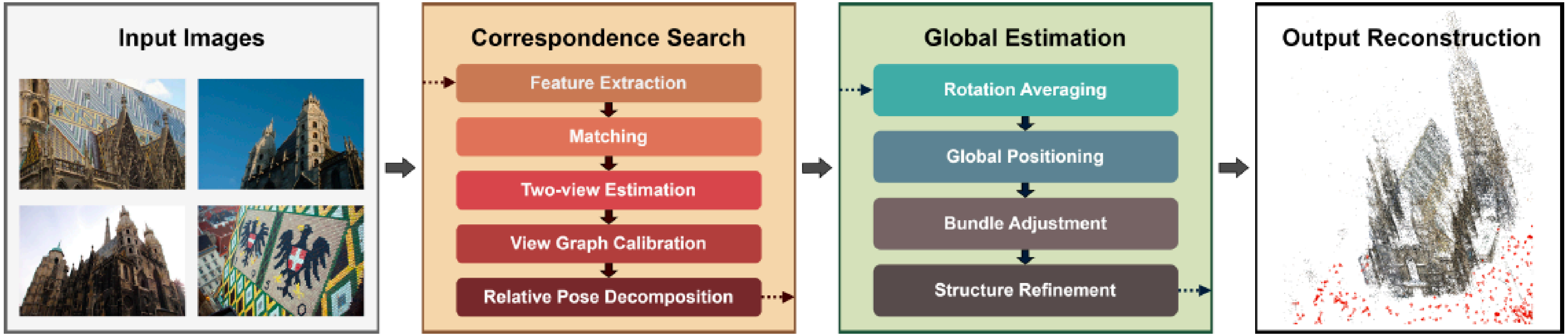


Özyeşil *et al.*, 2017, 2

Computer vision sketches of the problem … 2



Schönberger and Frahm, 2016, 2

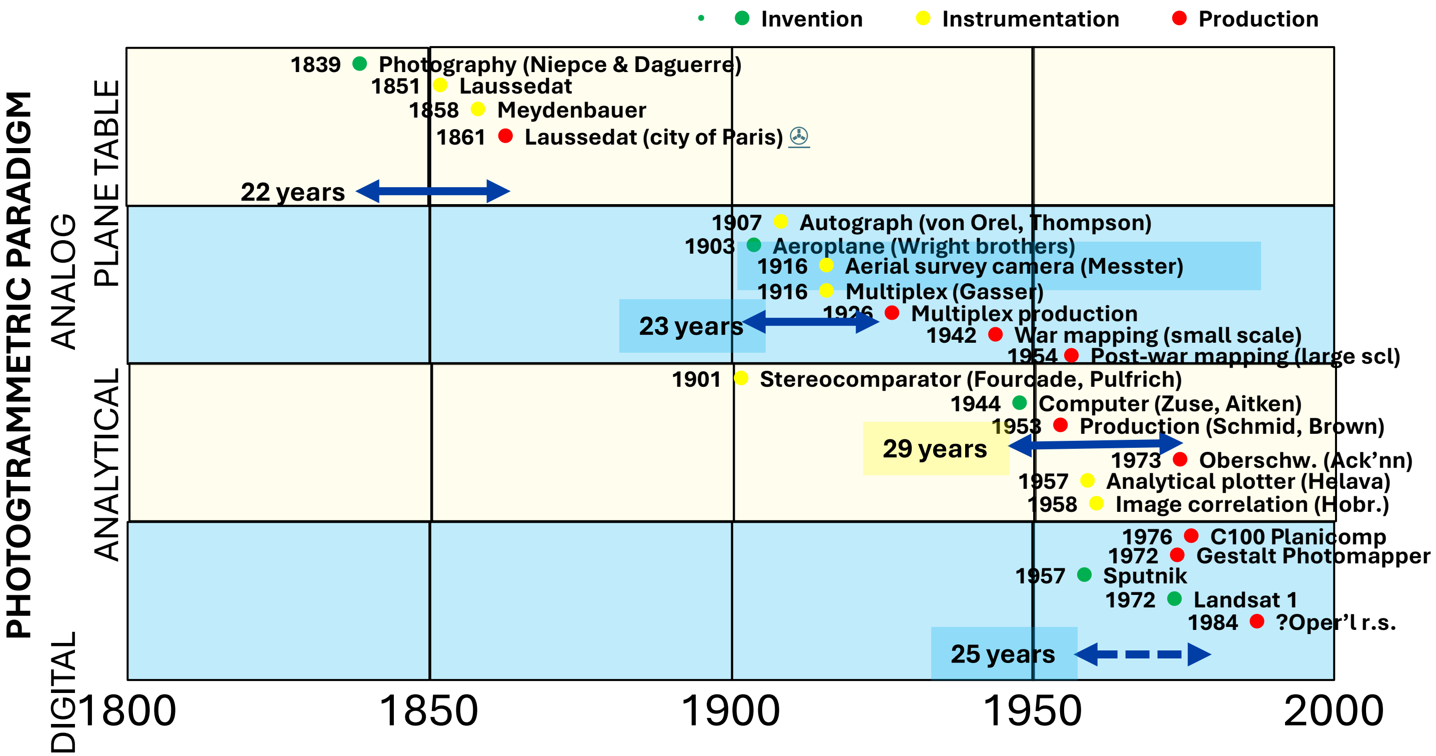


Pan et al., 2019, 7

Purpose of bundle adjustment

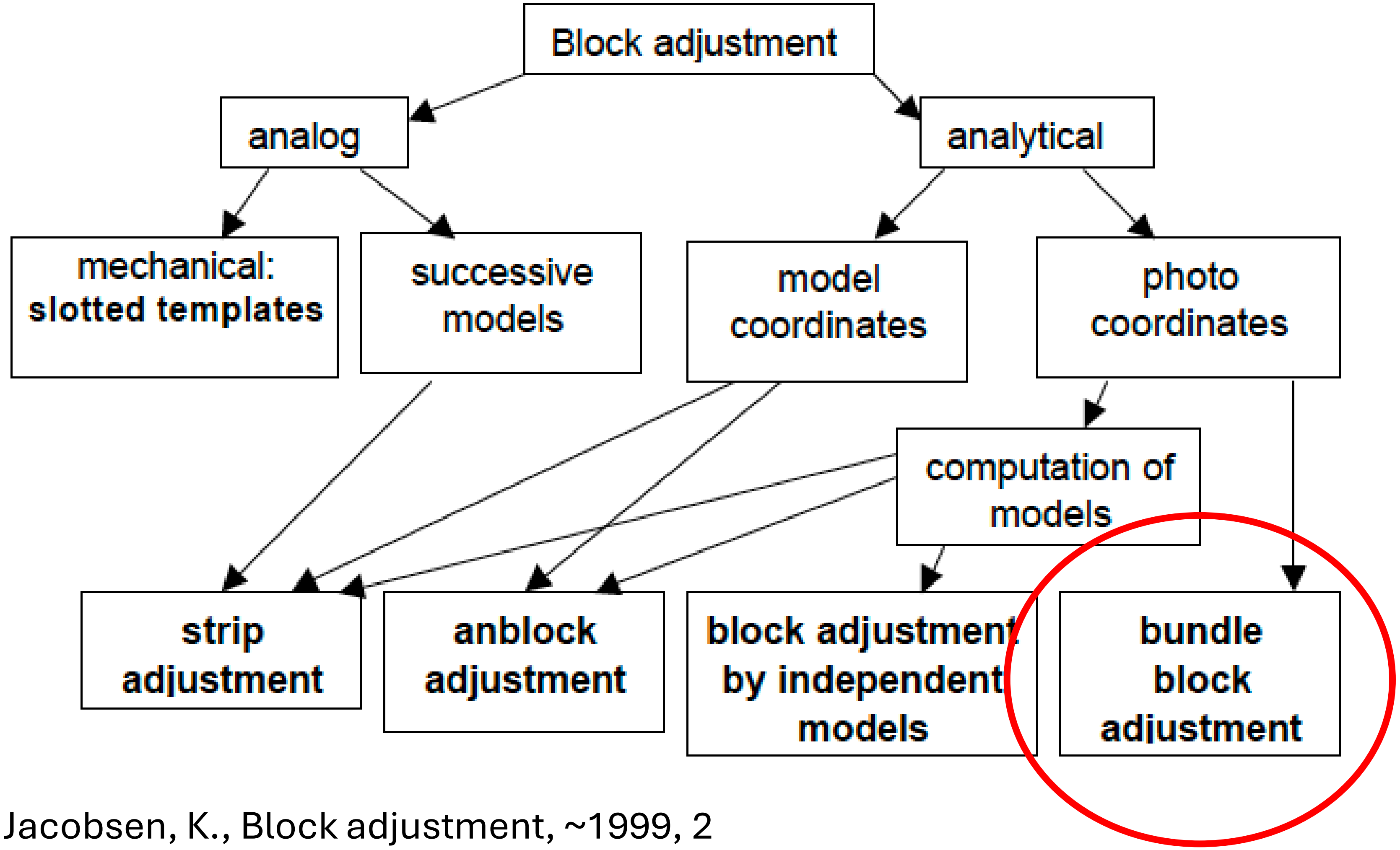
- The estimated coordinates of the ground points were used to set up individual stereo models for map compilation in stereoplotters
- This is still true today to some extent, but the enormously dense collections of ground points that are now derived can be used for other purposes, for example seeding algorithms for deriving elevation data by dense image matching; and sometimes the results are part of a trajectory computation, i.e. SLAM

Konecny's "paradigms of photogrammetry"



(After Konecny, *Photogrammetric Engineering & Remote Sensing*, 51 (7) 921, July 1985)

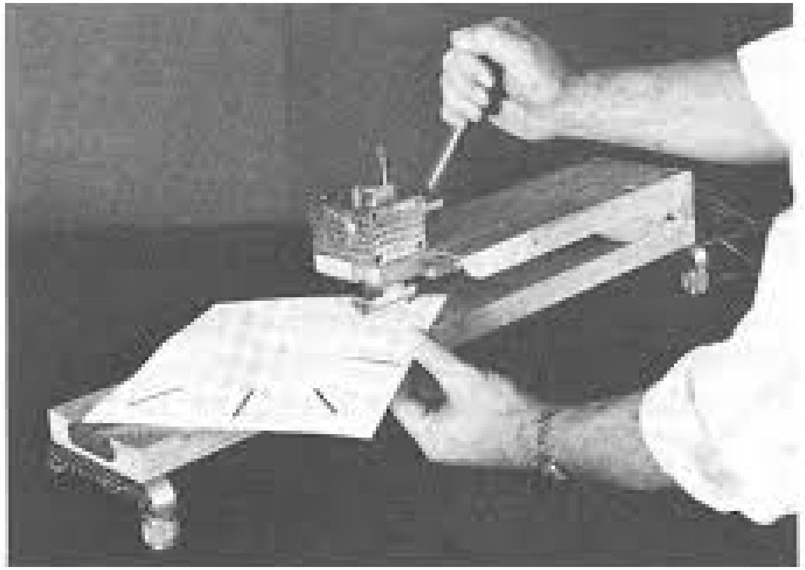
Taxonomy of triangulation

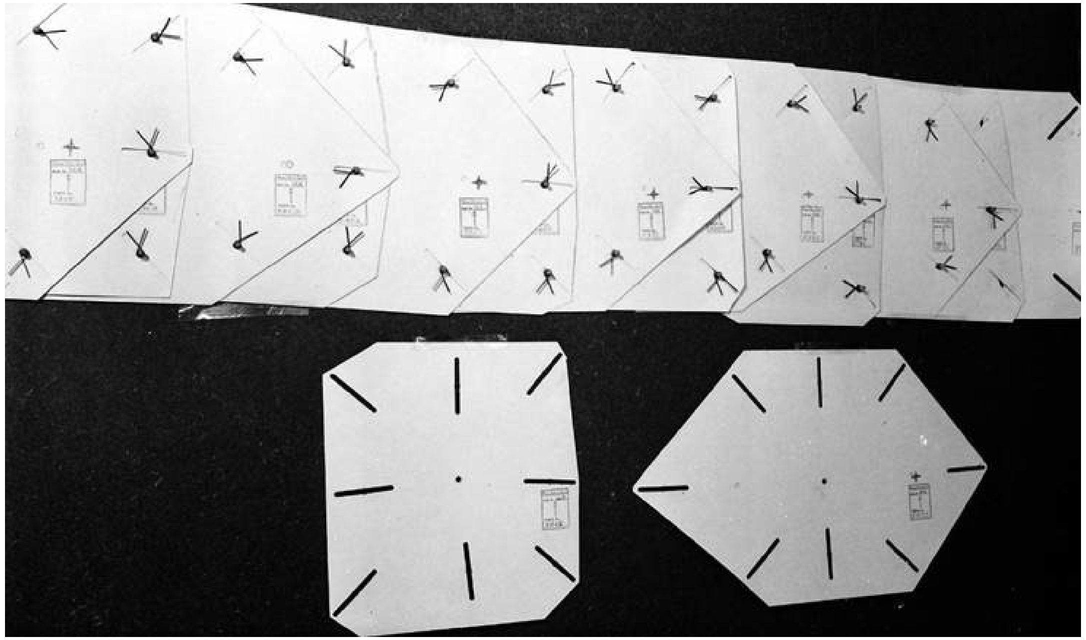


Jacobsen, K., Block adjustment, ~1999, 2

Slotted template assembly

- Late 1930s
- Slots cut in a piece of card or plastic overlaid on an aerial photograph – the “template”; the slots radiate from the fiducial center in the directions of the pass, tie and ground control points
- Templates are put together in an “assembly”, using pegs pushed through the slots in different templates that are related to common points





Slotted template assembly … 2

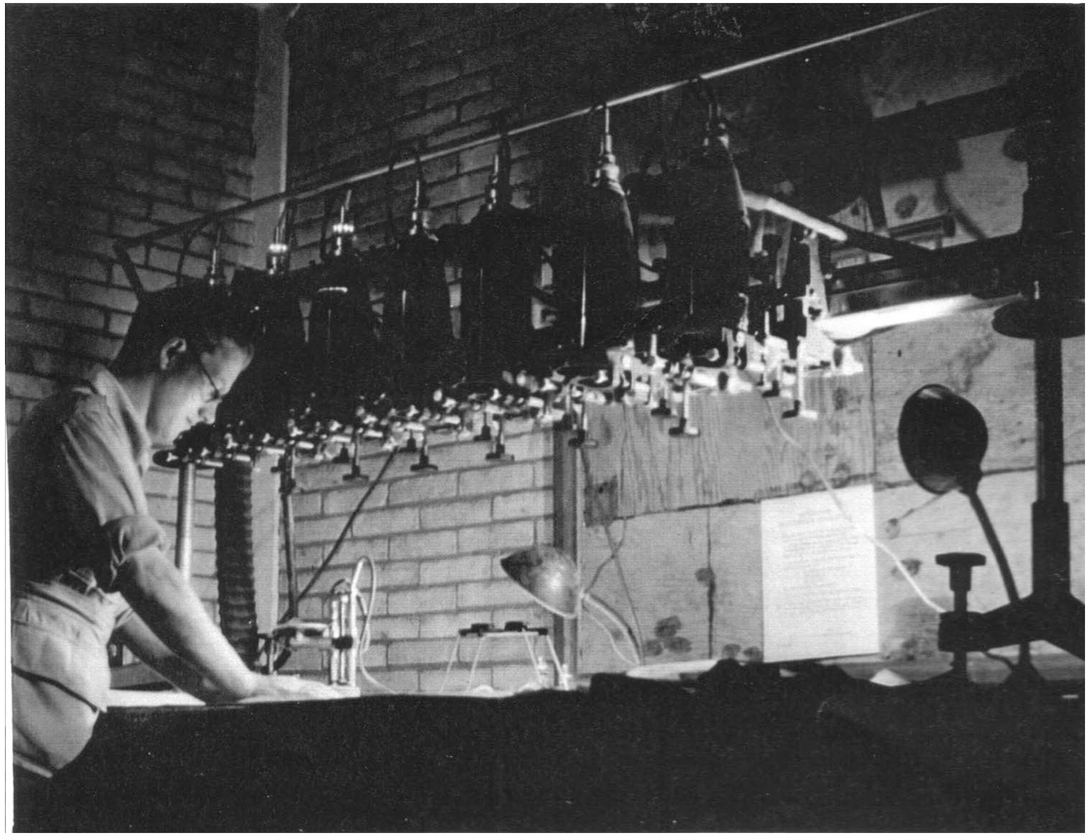
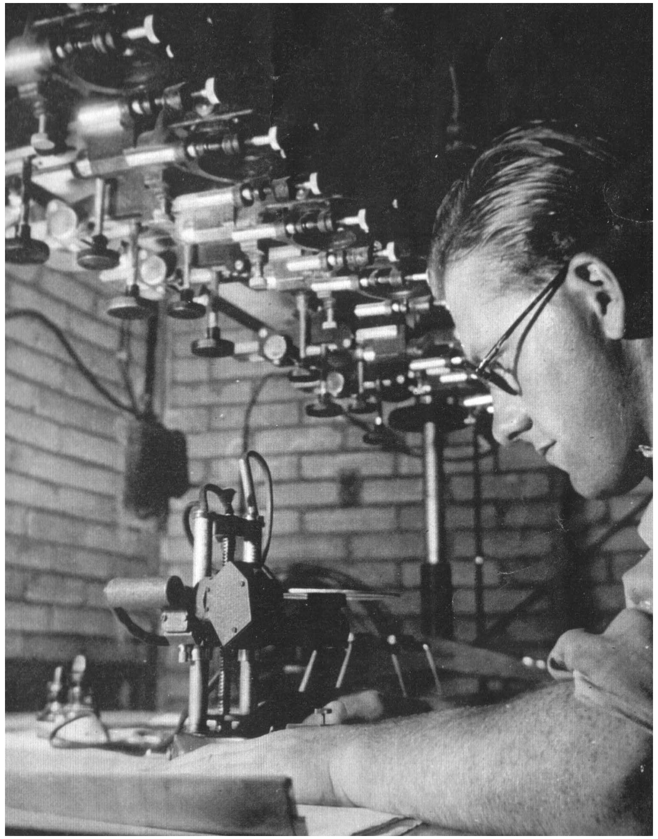
- The whole assembly is created on a board or floor, where the positions of ground control points are marked; the whole assembly settles, sometimes assisted by tiny steel ball-bearings, until the final positions of the pegs are the "adjusted" positions of the points of interest
- Pins are pushed through the pass and tie points pegs to mark the adjusted positions on the plot below
- The adjusted positions are then used for stereo compilation
- Also called "radial triangulation"

What happened next?

- Slotted templates were quickly left behind as instrumental methods developed
- Initially these were based on strips, either created on a photogrammetric instrument or by joining independent models together computationally
- The use of models and strips persisted even after bundle adjustment became the norm, as a device for early detection of gross errors

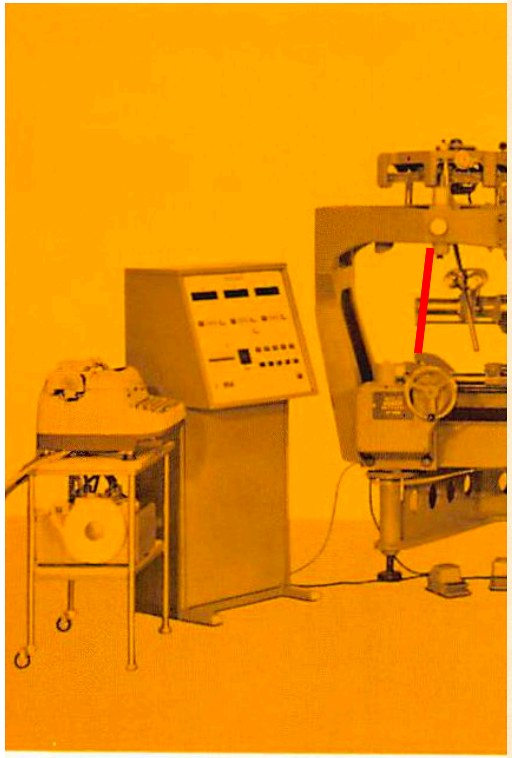
Strip formation on the instrument

- Fully analog process using longbar optical projection instrument, or first-order stereoplotter with “base-in/base-out”

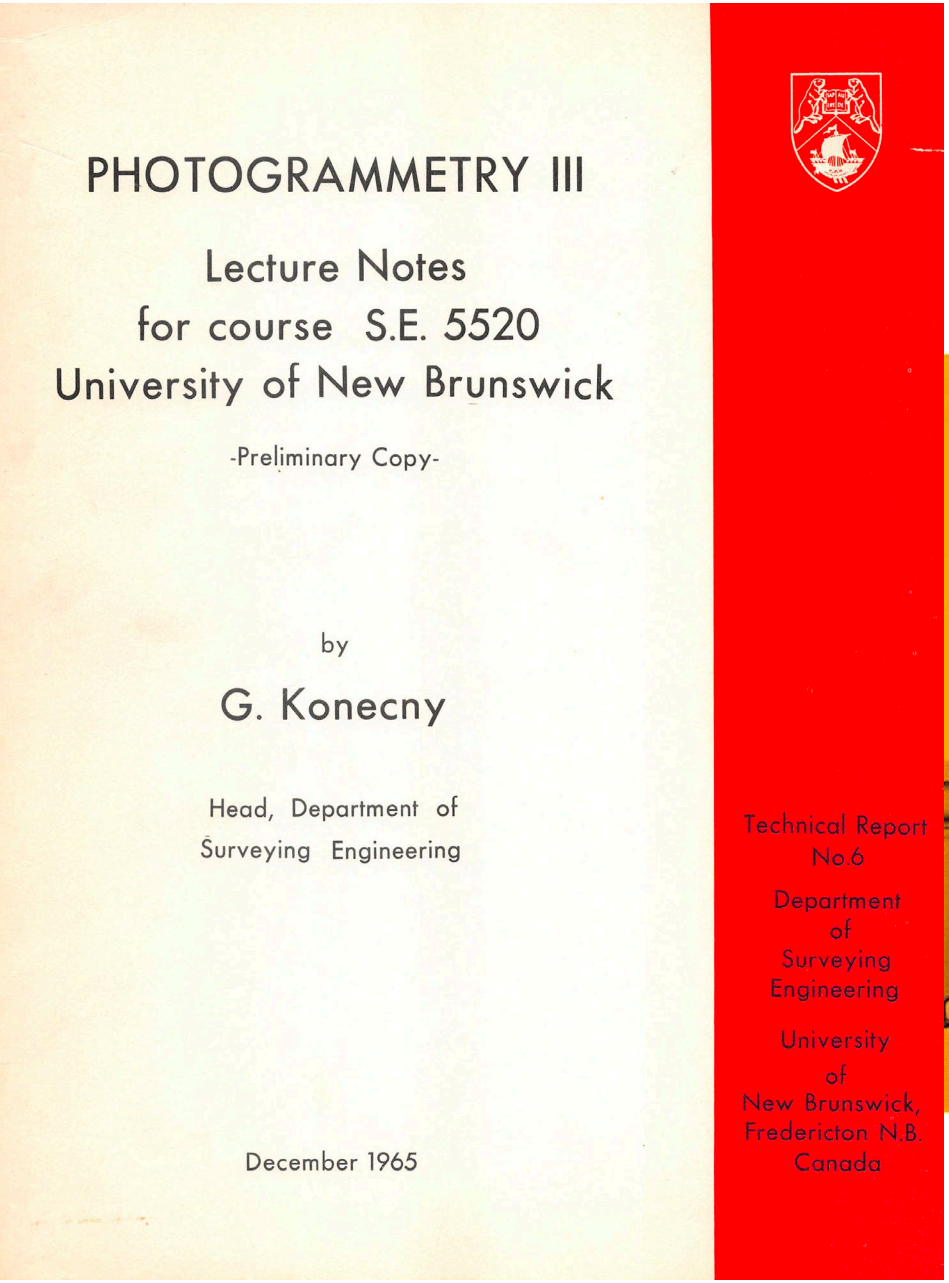



Base-in/base-out

- Stereomodel can be formed from the inward or outward parts of the diapositives and model coordinates measured, then photo 1 is removed, replaced with photo 3, model 2-3 observed etc. base-in/base-out



See also Colcord (1961)



PHOTOGRAMMETRY III

Lecture Notes
for course S.E. 5520
University of New Brunswick

-Preliminary Copy-

by

G. Konecny

Head, Department of
Surveying Engineering

December 1965

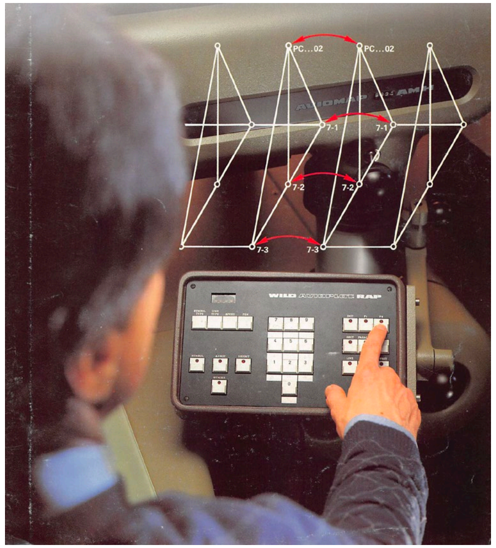
Technical Report
No.6

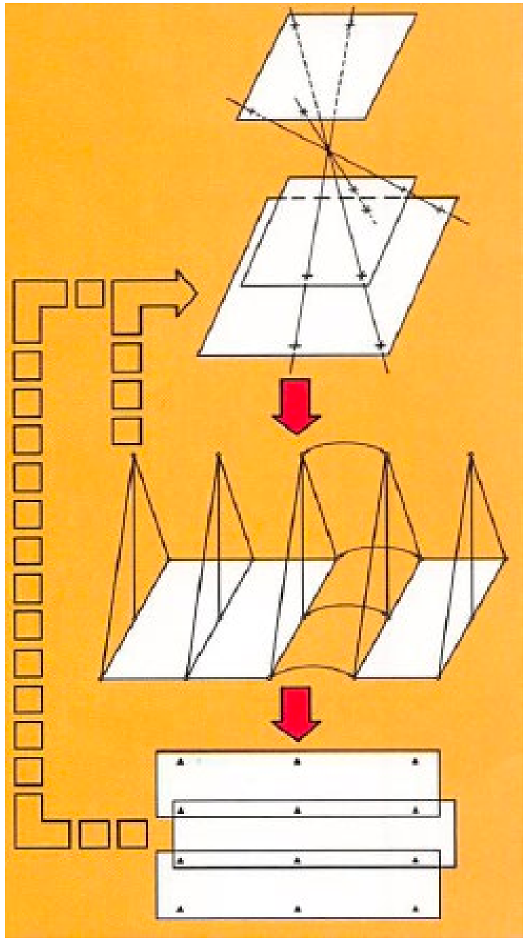
Department
of
Surveying
Engineering

University
of
New Brunswick,
Fredericton N.B.
Canada

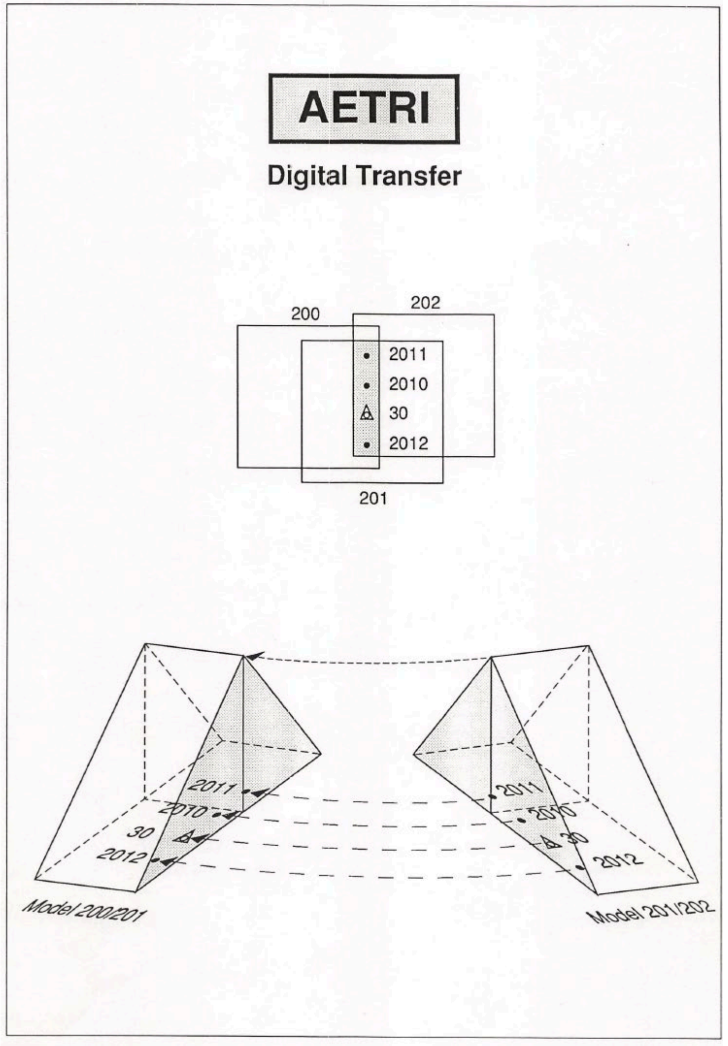
Connecting models computationally

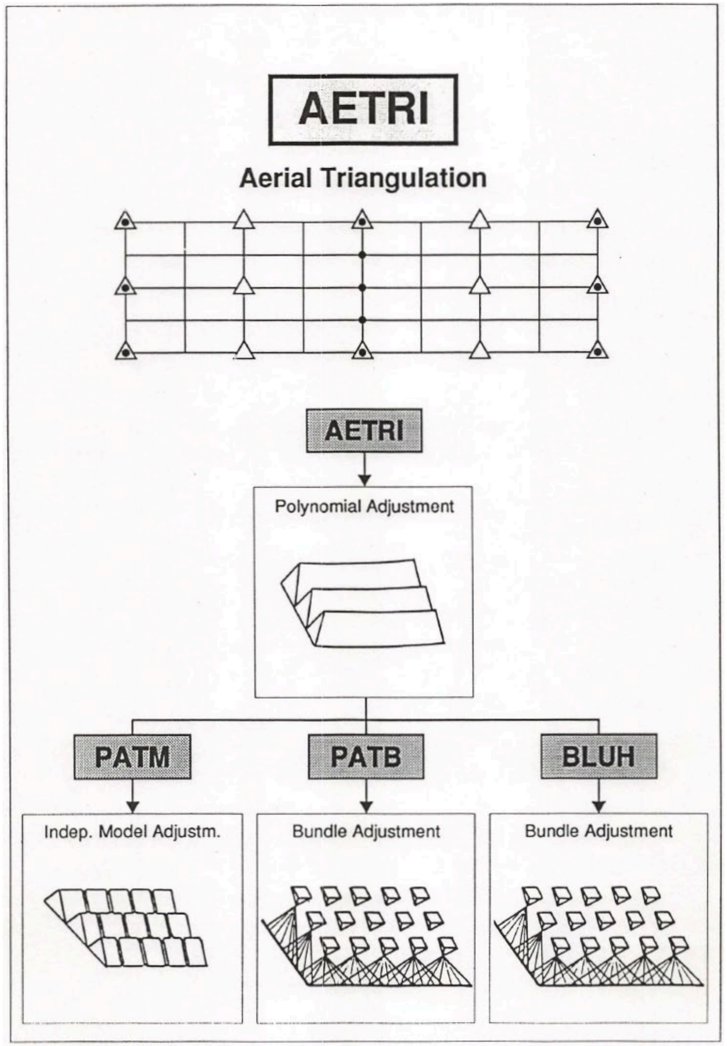
- Processes exist to determine the coordinates of the perspective center on a stereoplotter
- The model coordinates of the pass and tie points are measured stereoscopically in model space
- The models are joined by 3D similarity transformation
- Thus a strip is formed





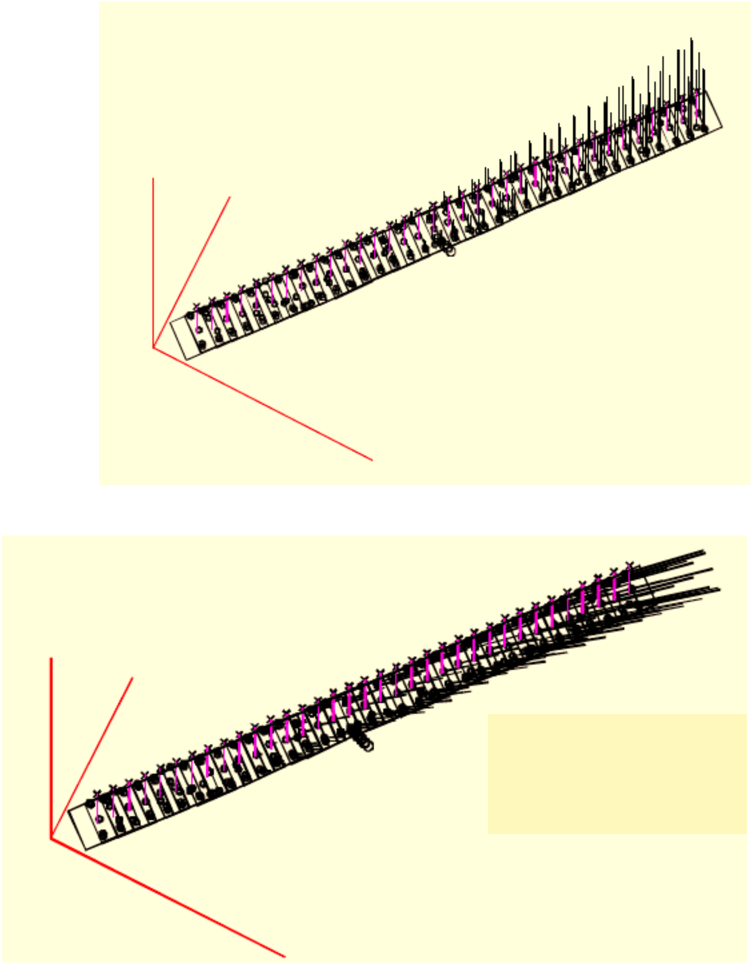
Kern sales literature, early 1980s





Waiting for bundles

- All the above approaches were focused on error detection and, more importantly, minimizing computer effort
- Considerable work by Ackermann and others, resulting in consensus that errors propagated along and across the strips such that they could be modeled by third- and second-order polynomials respectively
- As computers became available, strip adjustment software began to be developed, starting in the 1950s, e.g. G.H. Schut (NRC Canada) or J.E. Julia (Argentina)

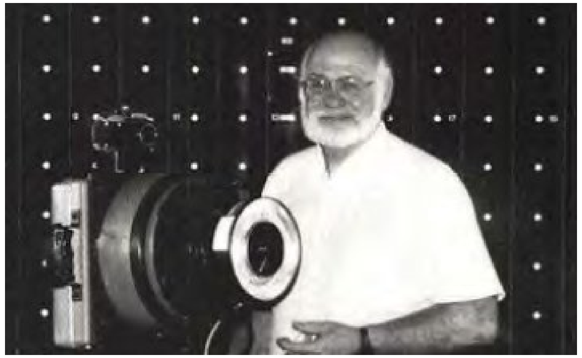


Jacobsen, 1999, 2-3

Transition to bundles

- Much early work was done with hand computation or early electronic calculators (Ordnance Survey in UK used a ”section” method (Proctor, 1962))
- Interfacing of minicomputers to analog stereoplotters came in early 1970s
- Analytical plotters invented 1957 by Uki Helava, but did not become widespread in non-military applications until Planicomp C100 in 1976
- Image coordinates could be measured on analog stereoplotters, but this was not popular; comparators and analytical plotters were the way to go

Transition to bundles … 2



- Bundle approach developed for US Air Force by Duane Brown and co-workers 1957-59
- Also Hellmut Schmid, Ballistic Research Laboratories, Aberdeen Proving Ground, for tracking satellites
- Concept of creating reduced normal equations, solving for the camera positions and attitudes, then back-substituting to solve for the ground points
  - For the typical photogrammetric block, the coefficient matrix was banded, with enormous numbers of null sub-matrices outside the band
- Initial software capable of handling tens of images
- Extended to close-range photogrammetry in 1960s
- Recursive partitioning to solve the normal equations came in 1967
- 1000 photos feasible by late 1960s
  - Or was that wishful thinking?

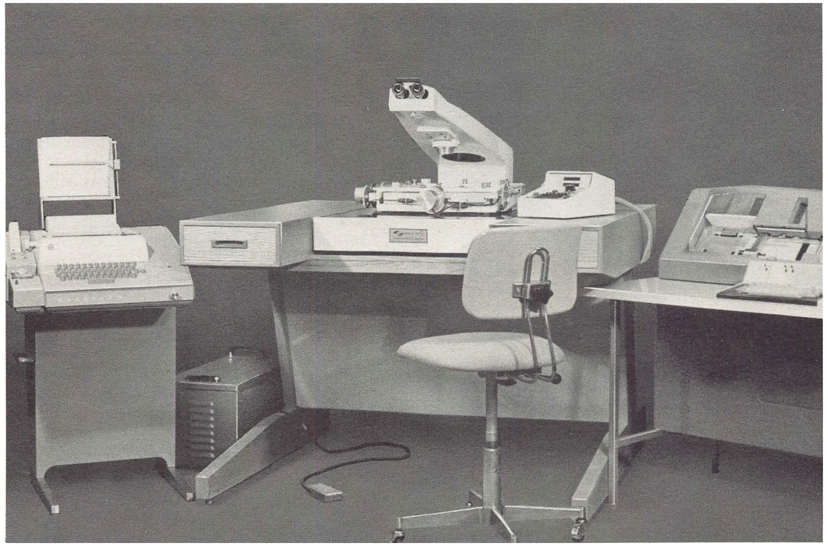
Observing the image coordinates

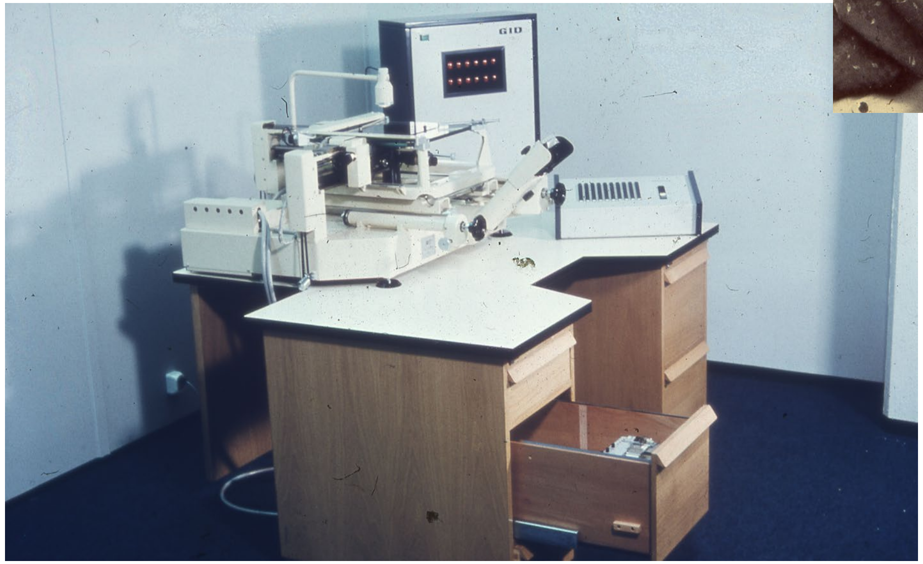
- Image coordinates could be derived from the model coordinates measured on an analog stereoplotter, but this was uncommon
- Monocomparators and stereocomparators
- Analytical stereoplotters
- Digital photogrammetric workstations and general-purpose computers and image processing systems

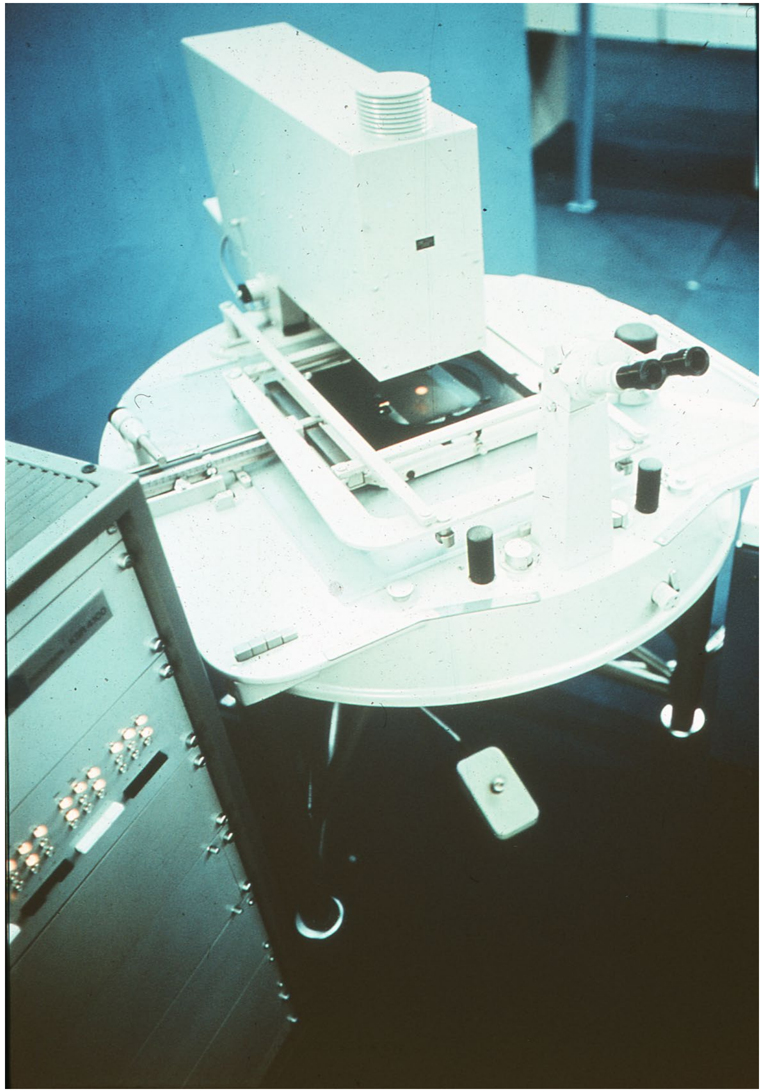
Monocomparators

- Instruments to measure image coordinates on one photo
- Viewing could be monoscopic or stereoscopic

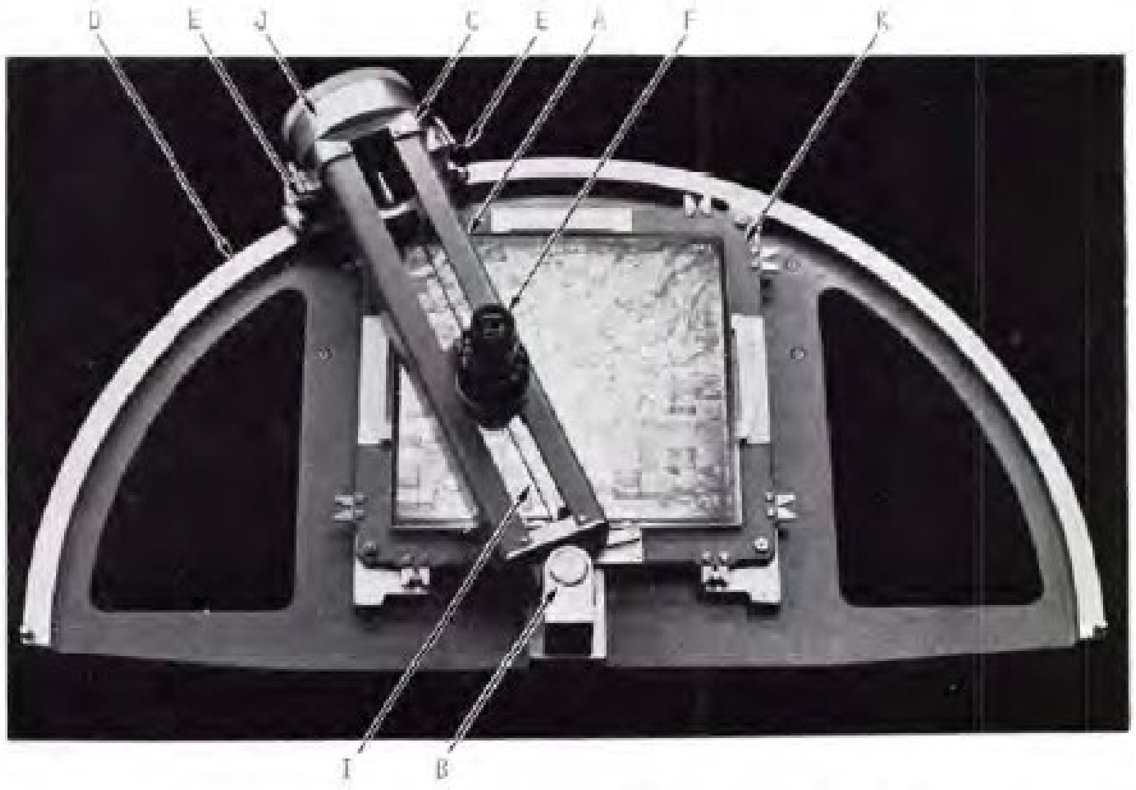
Monocomparators

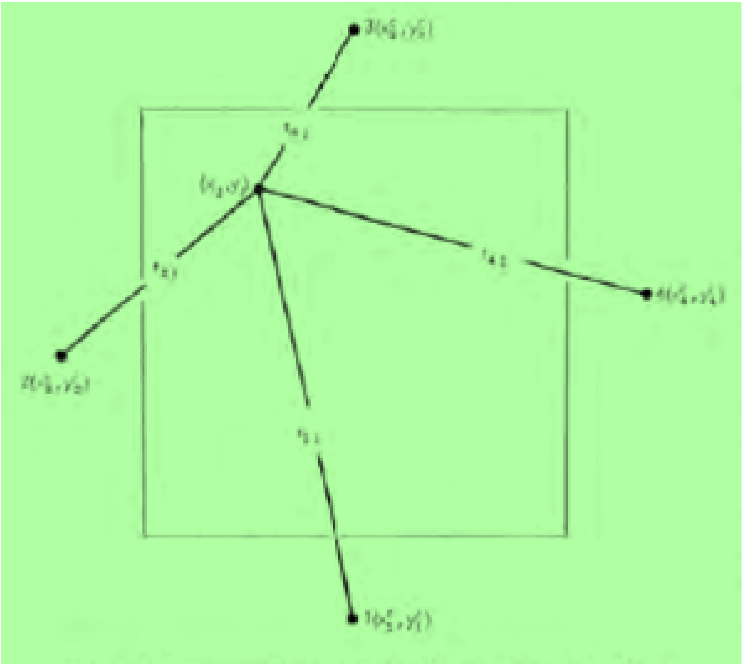






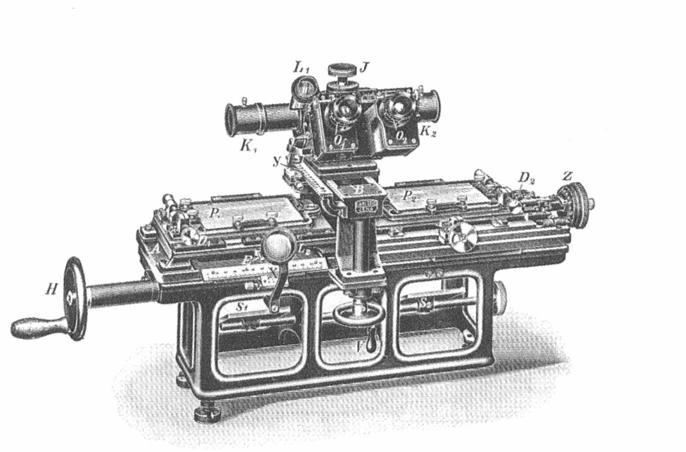
Duane Brown Multilaterative Monocomparator



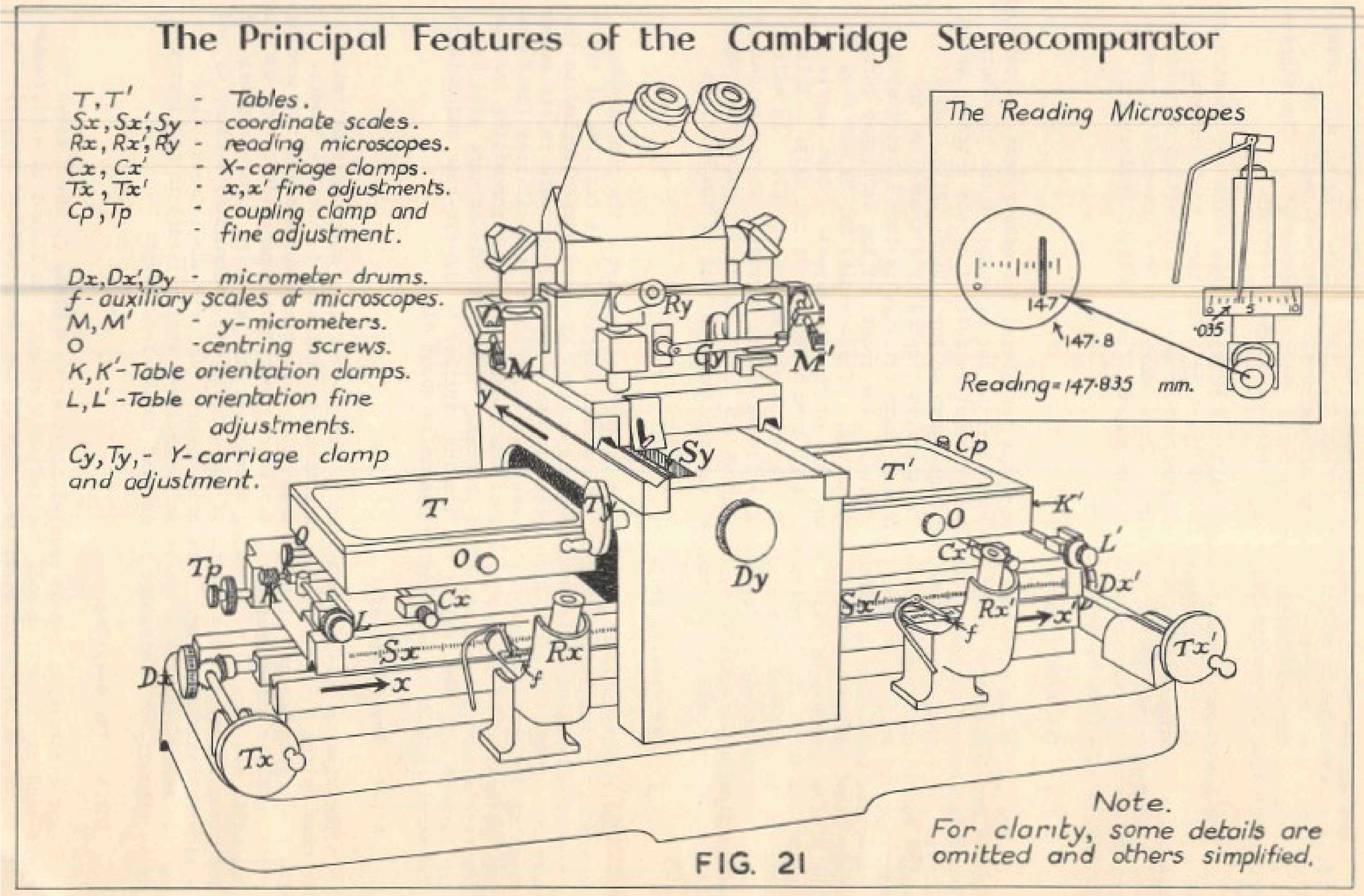


Stereocomparators

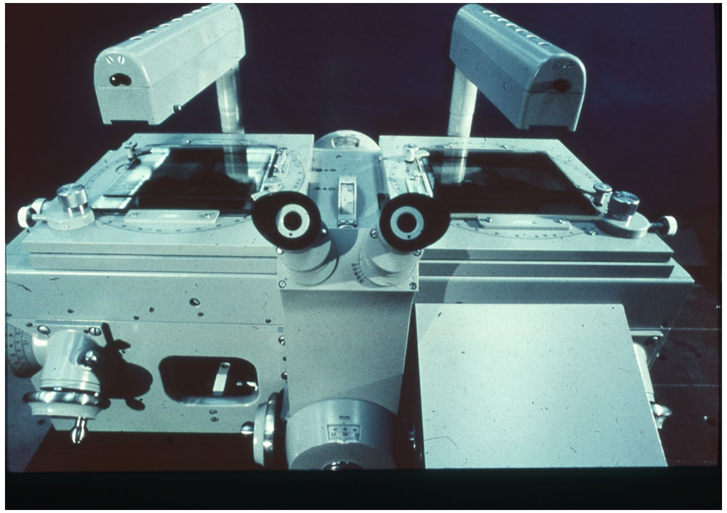
- Measuring coordinates of a point in two or more photos
- Viewing was stereoscopic
- Goes back to Pulfrich's Zeiss Jena stereocomparator of 1901
  - Stereoscopic vision and floating marks
  - Abbe's comparator principle

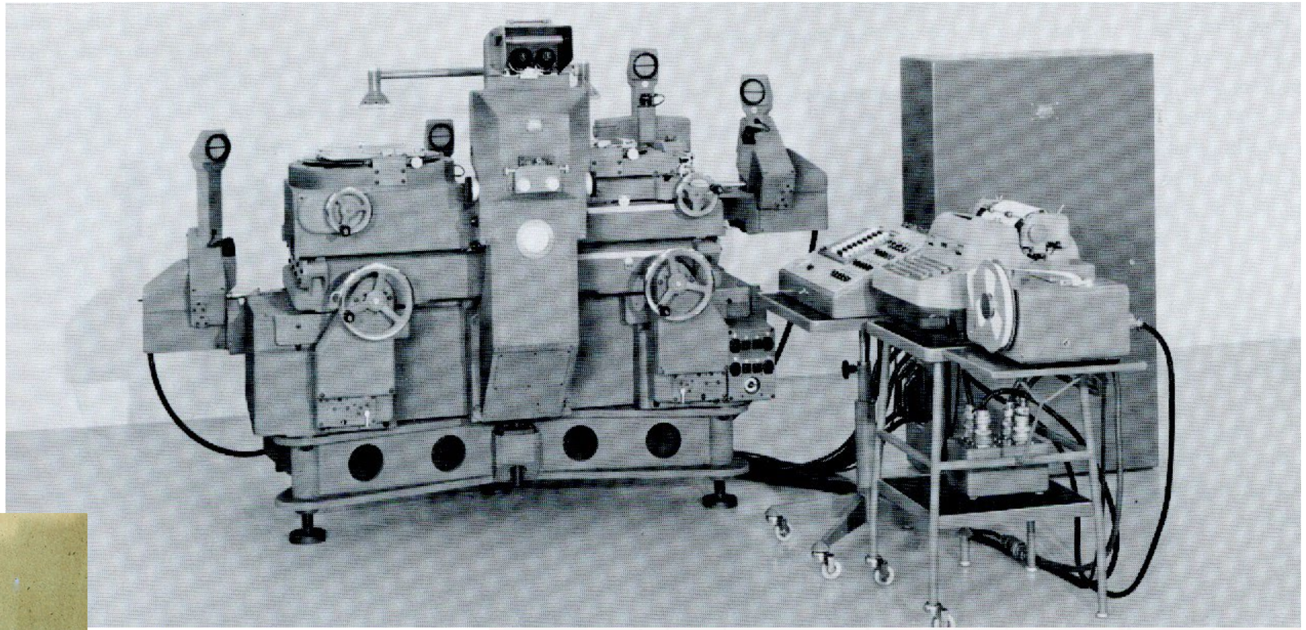


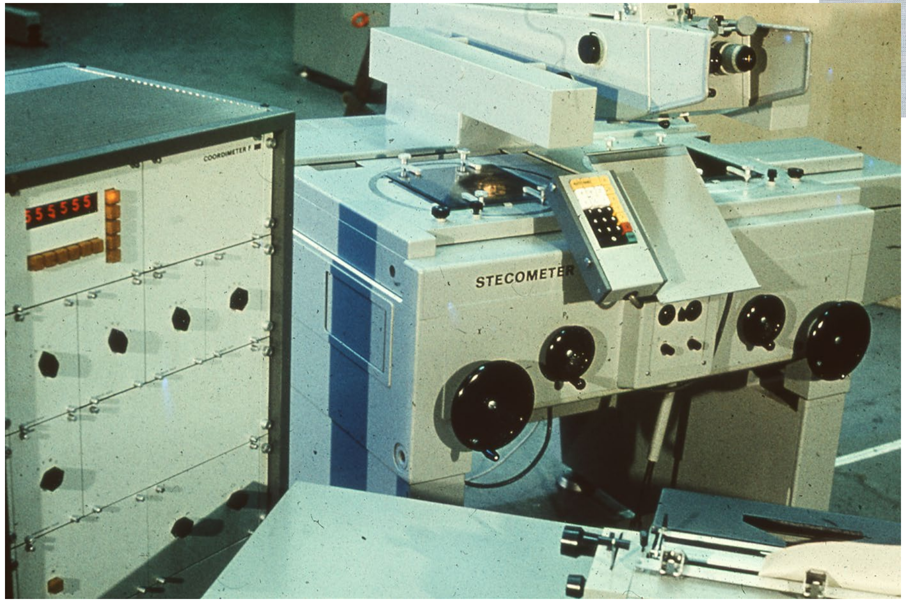
Cambridge stereocomparator



Stereocomparators



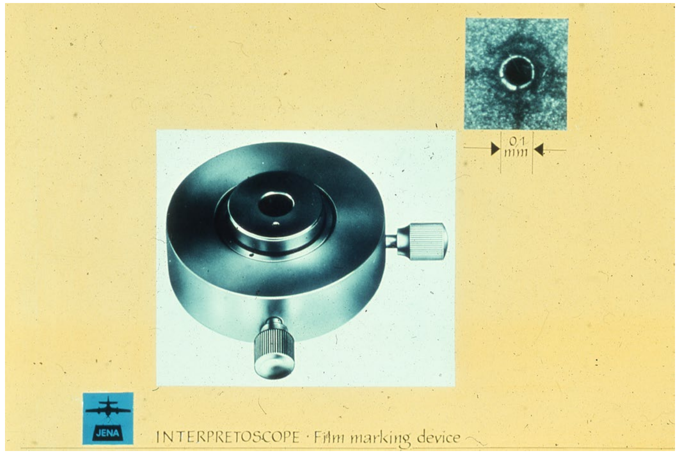


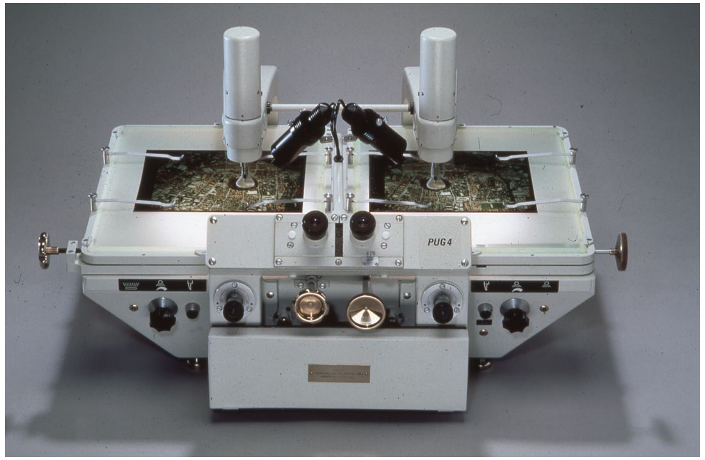


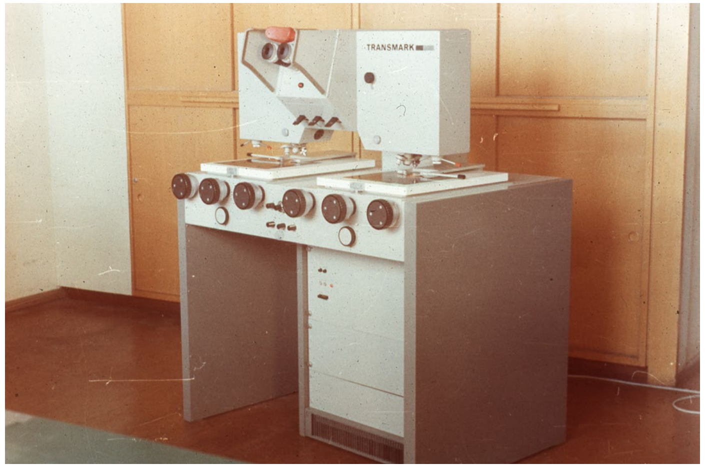
Point marking and transfer devices

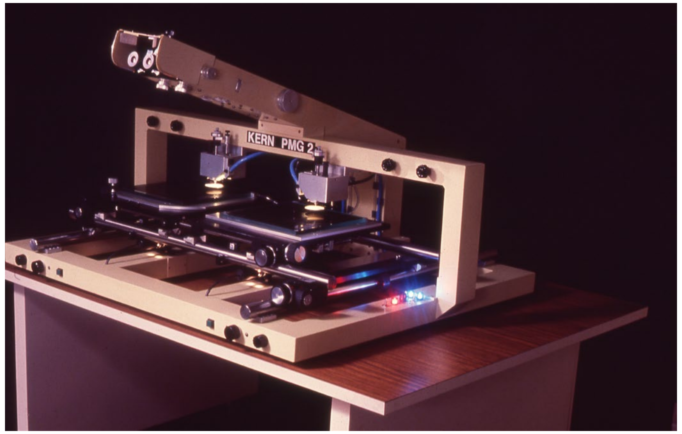
- The points to be measured could be
  - Targeted on the ground, but then you had to fly exactly so that they would be in the right places to be pass or tie points
  - Natural points, carefully documented on paper forms – oh, so laborious!
  - Artificial points, marked on the emulsion of the diapositive
    - Forcible indentation
    - Drilling
    - Burning by laser
- Pass points were marked between images, in the triple overlaps when the standard sidelap of 50% was used
- Tie points between strips
- Typically points in the von Gruber location, usually one, sometimes 2-3

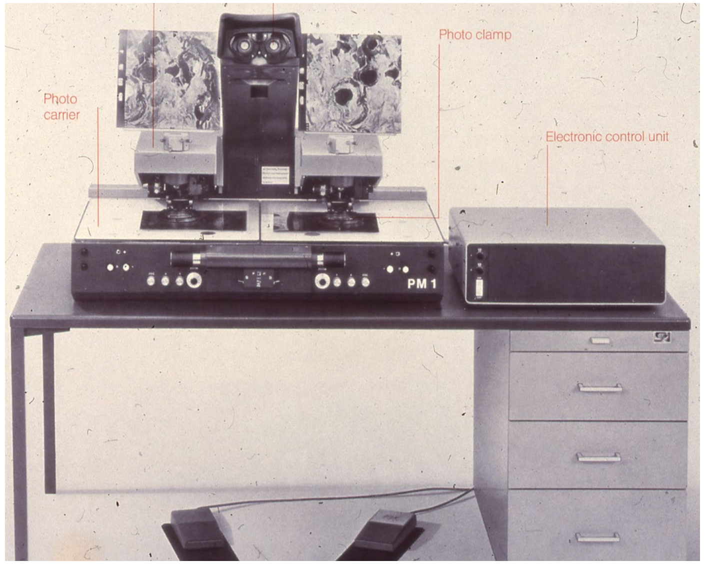
Point marking and transfer devices



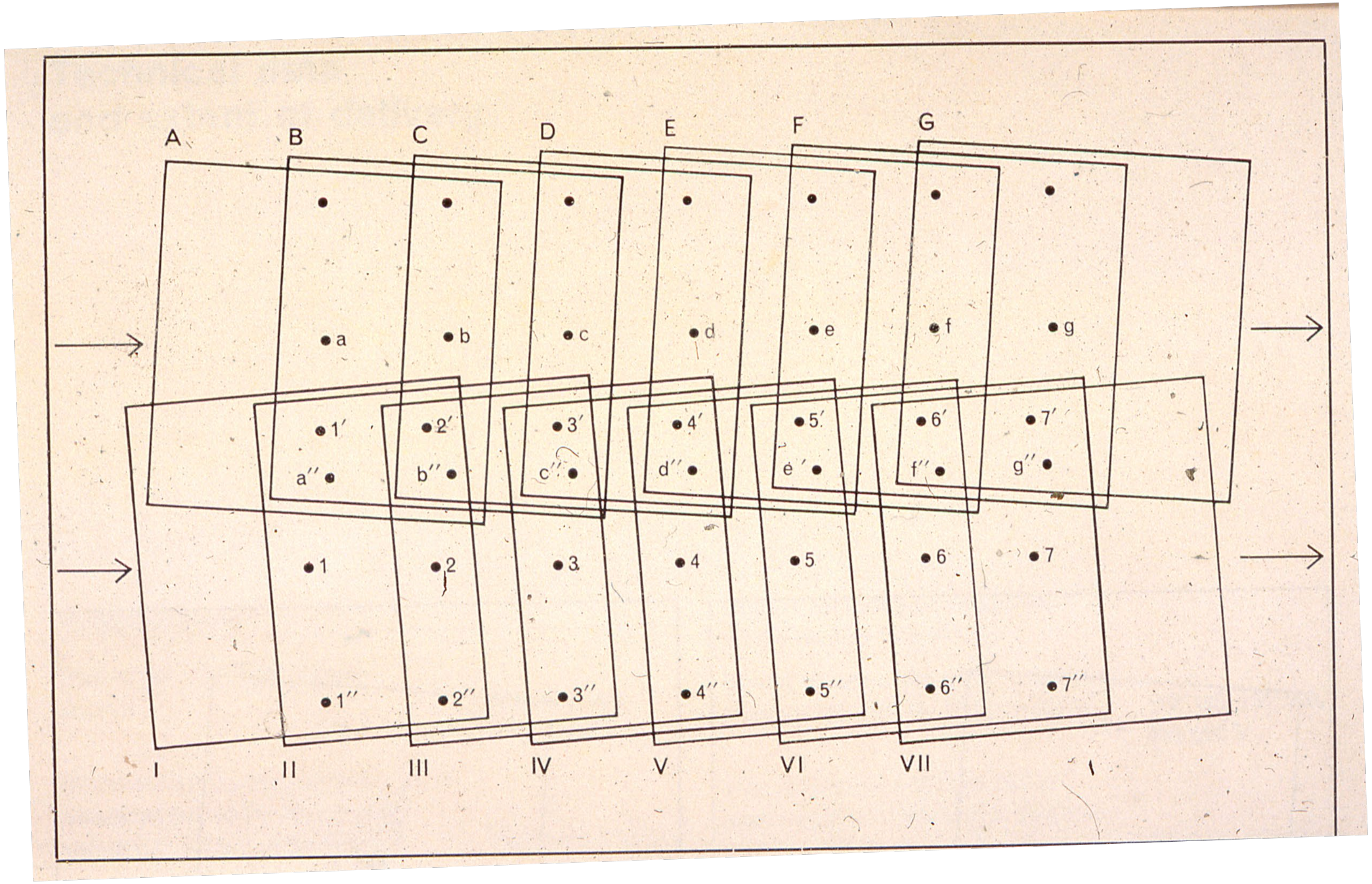








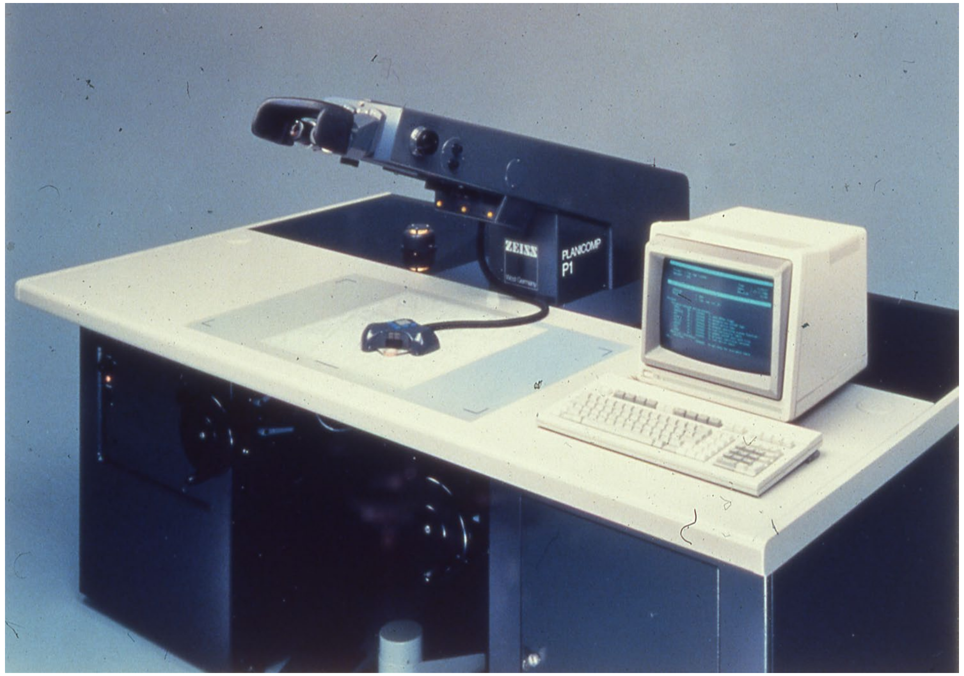
Points marked in von Gruber locations



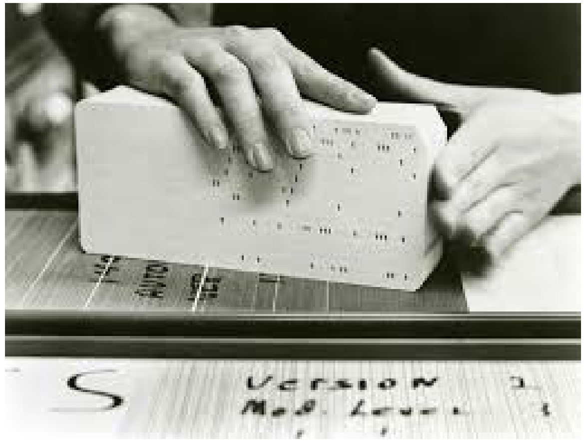
Analytical stereoplotters

- High-end ones best for observing image coordinates for triangulation – Wild AC1 rather than BC1, Leica SD3000 rather than SD2000 etc.
    - Precision ~1 μm
- Analytical plotters recorded the image coordinates of the pass and tie points, which could therefore be revisited and transferred whenever the photos were put back in the instrument: thus point marking no longer necessary, though some people used it

Analytical plotters … 2



Computers open the three doors



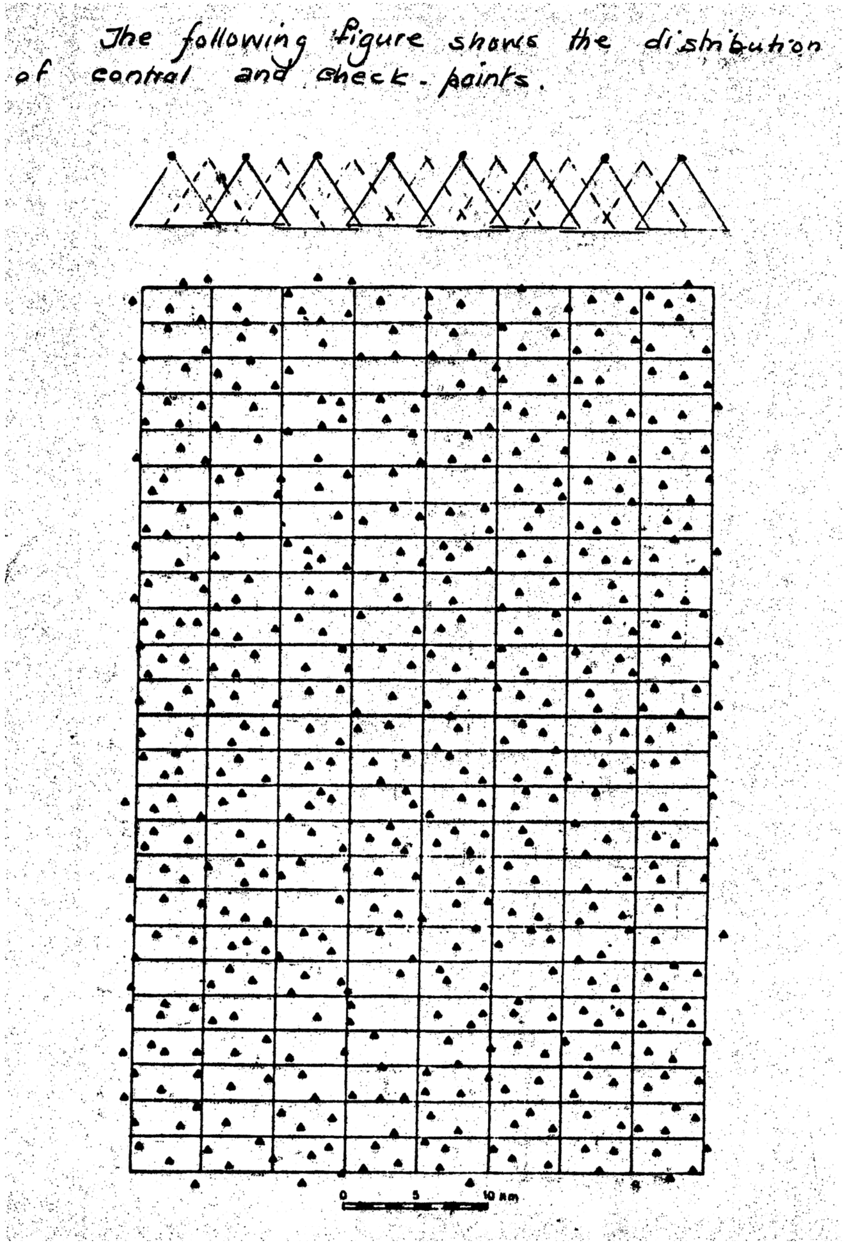
- Mainframe computers became readily available to photogrammetric researches (and practitioners, especially in government agencies) by the late 1960s if not before
  - Early adjustment packages were fed with punch cards!
- Three approaches characterized block adjustment in aerial triangulation:
  - Polynomial adjustment of strips (Schut, Julia *et al*.)
  - Block adjustment of independent models using similarity transformations or similar (van den Hout – Anblock; Ackermann, Ebner and Klein – PAT-M43/PAT-M7; R.A.J. Blais – SPACE-M)
  - Bundle adjustment (Duane Brown led the way)

Accuracy tests

- Many tests of block adjustment were performed and thousands of reports and papers written
- Test ranges were constructed with marked control points, which were accurately surveyed using traditional land surveying methods
- We're all heard of Vaihingen, but earlier examples were Oberschwaben and Wittenbach
- Some of the reports were very authoritative, e.g. those from OEEPE, the precursor of EuroSDR
- Helped develop guidelines for the quantity and placement of ground control points

Test range Oberschwaben

- 40 x 62.5 km
- 15 strips of 26 photos, 1:28,000; both wide and super-wide angle, total = 780
- 544 surveyed points with 80 x 80 cm premarking, 480 also known in elevation
- Cost >200,000 DM



Bundle adjustment compared to the others

- Perhaps in the early days, bundle adjustment was seen as more of a “black box”, less intuitive than strips or models (given that most people in the science at that time had some teaching or experience on stereoplotters)
- Yet people understood that bundle adjustment was the most rigorous representation of reality – what could be more beautiful or more easily understood than the collinearity equations?
- Bundle adjustment required more computer power – in addition to the XYZ coordinates of all the unknown points, there were 6 unknowns per image, as opposed to 7 per model

Why did bundle adjustment not give better results than independent models

- Some discerning experts in the community noted that bundle adjustment did not always generate notably better results than independent models
- Why?
- Eventually, it dawned: there were systematic image errors present that were not embodied in the camera calibration, yet somehow were being compensated in the process of model formation on a stereoplotters, e.g. platen non-flatness
  - Calibration conditions differ from flight conditions!
- These systematic errors meant that the rigorous calibration of the cameras, performed on specially built optical equipment, wasn't quite enough

Additional parameters

- The way forward was to add “additional parameters”, i.e. further terms to the collinearity equations to represent small systematic errors in the image coordinates
- Concept introduced 1964 and implemented by 1968: accuracy improvements of 2-10x; Brown and co-workers heavily involved
- Even until the present day the formulation of these additional parameters has been a subject of intense debate and experimentation:
  - Small changes to the position of the principal point or the principal distance
  - Physical, along the lines of optical calibration, e.g. power series for radial lens distortion
  - General purpose polynomials, not modeling any particular reality
  - Exotic polynomials, wavelets etc.
  - Best are those with minimum correlation between parameters
  - Karsten has 207 of them at the latest count!

Brown in 1974 and 1976

- Two critically important review papers
  - The 1976 update was entitled, “The bundle adjustment – progress and prospects”
- Laid out the mathematics of creating normal equations with two sets of unknows – camera parameters and ground points
- Reviews the structure of the normal equations and the generation of reduced normal equations to solve for the camera parameters
- Used sparseness (reduced normals were banded) and successive overrelaxation, but the latter was replaced by recursive partitioning
- Then he explored self-calibration, with an example (auxiliary sensors treated as part of this) – “Brown parameters”
  - Required recursive partitioning of banded and bordered system
  - First application to aerial photogrammetry was by Bauer and Müller (1972), over Oberschwaben – spectacular results!

Thompson in 1976

- Address to the Photogrammetric Society, 16 March 1976, published posthumously
- Snapping at the heels of nomenclature
- "The so-called bundle method is not the most commonly adopted method for. calculation, but it is probably the simplest to describe." (Thompson, 1976, 714)
- "Attempts are being made to obtain the interior orientation elements from measures made "on the job". We are of the opinion that these attempts are not realistic and will not result in improved accuracies. It would be more profitable to modify laboratory conditions to simulate those likely to be met during flight (particularly those concerning temperature and pressure)." (Thompson, *ibid*., 715)
- Believed that independent models could give superior results owing to the rigorous relative orientation used in model formation

Computational aspects

- Let's start with the observation equations, for point j in photo i:
- It looks as if this formulation mixes together the unknown coordinates, orientation parameters (position and attitude) and additional parameters, so that the model is of the form:

$$f(\boldsymbol{x},\ell) = 0$$

where f is a nonlinear function, and $\boldsymbol{x}$ is the vector of unknown parameters and coordinates and ℓ is the vector of observed image coordinates

- This is solved iteratively, starting from initial approximations $\boldsymbol{x}^0$:

$$\Delta \boldsymbol{x} = [B^T(ATWA)^{-1}B]^{-1}B^T(ATWA)^{-1}f^0$$

Where B and A are matrices of partial derivatives with respect to the unknown parameters and observations respectively and f^0 is the function evaluated at the initial values $\boldsymbol{x}^0$ (some authors use A and B!) and W, called a weight matrix, is the inverse of the covariance matrix of the observations

- Poor initial values lead to risk of divergence, or convergence on a local minimum in the cost function

Computational aspects … 2

- To save hours of typing into the Word equation setter-upper, we reproduce the equations from the superb exposition by Förstner et al. in the 6th edition of the *Manual of Photogrammetry*
- This done by writing simple linear equations equating each unknown orientation parameter, ground control point coordinate and additional parameter to its initial value and adding further weight matrices to reflect the quality of the initial values
- For example, we have quite good values for the orientation parameters from the GNSS/IMU and for the ground control points from the professional land surveyor who measured them

Formulation from *Manual of Photogrammetry*

- Observation equations:

$$F_{ij} = \begin{bmatrix} F_l \\ F_s \end{bmatrix}_{ij} = \begin{bmatrix} g\left(\hat{l}_{ij}, \hat{s}_{ij}, \hat{G}_j, \hat{\Omega}_i\right) \\ h\left(\hat{l}_{ij}, \hat{s}_{ij}, \hat{G}_j, \hat{\Omega}_i\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} F_{11} \\ F_{12} \\ \vdots \\ F_{1m(1)} \\ F_{21} \\ F_{22} \\ \vdots \\ F_{2m(2)} \\ \vdots \\ F_{n1} \\ \vdots \\ F_{nm(n)} \end{bmatrix}$$

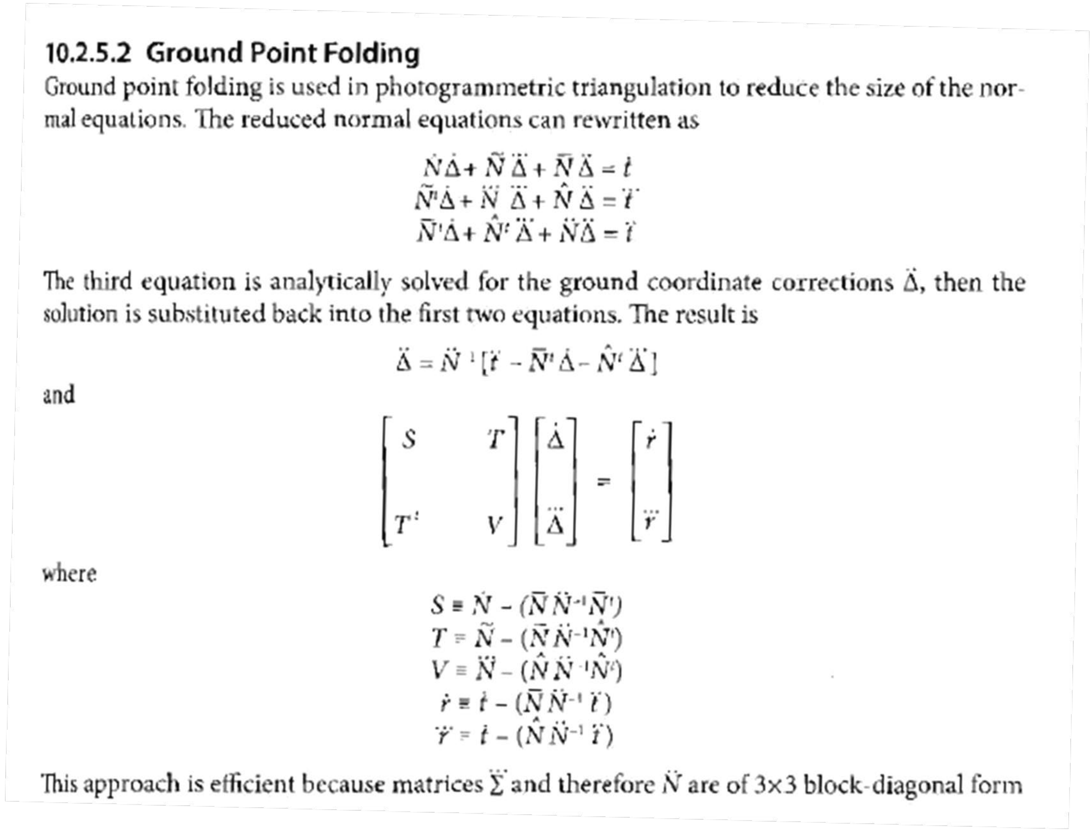
- Linearization:

$$F^O + \frac{\partial F}{\partial L} \Delta L + \frac{\partial F}{\partial X} \Delta X = 0$$

$$\hat{L} = L^{OBS} + v = L^0 + \Delta L$$
$$\hat{X} = X^0 + \Delta X$$

Formulation from *Manual of Photogrammetry* … *7*

- The solution of this system is a paradise for matrix algebra fans
- The system is a partitioned one and the normal equations can be written as



10.2.5.2 Ground Point Folding

Ground point folding is used in photogrammetric triangulation to reduce the size of the normal equations. The reduced normal equations can rewritten as

$$\dot{N}\dot{\Delta} + \tilde{N}\dddot{\Delta} + \bar{N}\ddot{\Delta} = \dot{t}$$
$$\tilde{N}^t\dot{\Delta} + \dddot{N}\dddot{\Delta} + \hat{N}\ddot{\Delta} = \dddot{t}$$
$$\bar{N}^t\dot{\Delta} + \hat{N}^t\dddot{\Delta} + \ddot{N}\ddot{\Delta} = \ddot{t}$$

The third equation is analytically solved for the ground coordinate corrections $\ddot{\Delta}$, then the solution is substituted back into the first two equations. The result is

$$\ddot{\Delta} = \ddot{N}^{-1}[\ddot{t} - \bar{N}^t\dot{\Delta} - \hat{N}^t\dddot{\Delta}]$$

and

$$\begin{bmatrix} S & T \\ T^t & V \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \dddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{r} \\ \dddot{r} \end{bmatrix}$$

where

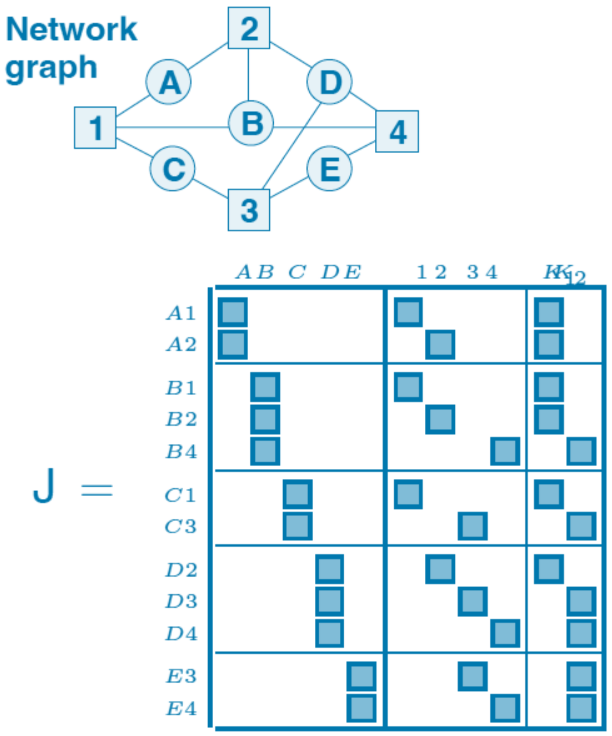
$$S \equiv \dot{N} - (\bar{N}\ddot{N}^{-1}\bar{N}^t)$$
$$T \equiv \tilde{N} - (\bar{N}\ddot{N}^{-1}\hat{N}^t)$$
$$V \equiv \dddot{N} - (\hat{N}\ddot{N}^{-1}\hat{N}^t)$$
$$\dot{r} \equiv \dot{t} - (\bar{N}\ddot{N}^{-1}\ddot{t})$$
$$\dddot{r} \equiv \dddot{t} - (\hat{N}\ddot{N}^{-1}\ddot{t})$$

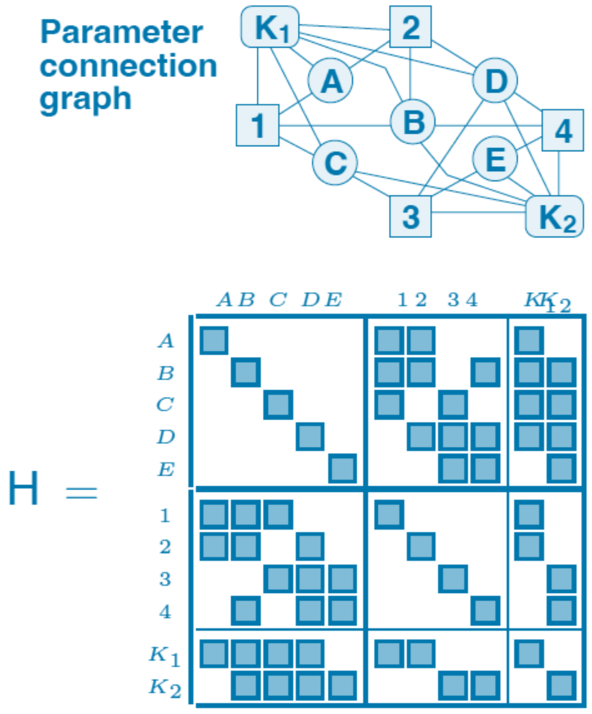
This approach is efficient because matrices $\ddot{\Sigma}$ and therefore $\ddot{N}$ are of 3×3 block-diagonal form

Computational aspects … 3

- We perform various back-substitutions so that we create a system of normal equations for the image parameters only, called the reduced normal equations (sometimes called Schur complement)
- This system is enormous, but not nearly as big as it would be if we didn't do this reduction
- Also, the reduced normal equations matrix has a very special characteristic: it is banded and bordered, with lots of null sub-matrices, so photogrammetrists, computer vision folk and mathematicians spent many happy decades optimizing the solution methods

Very special matrices





Triggs et al., 1999, 318

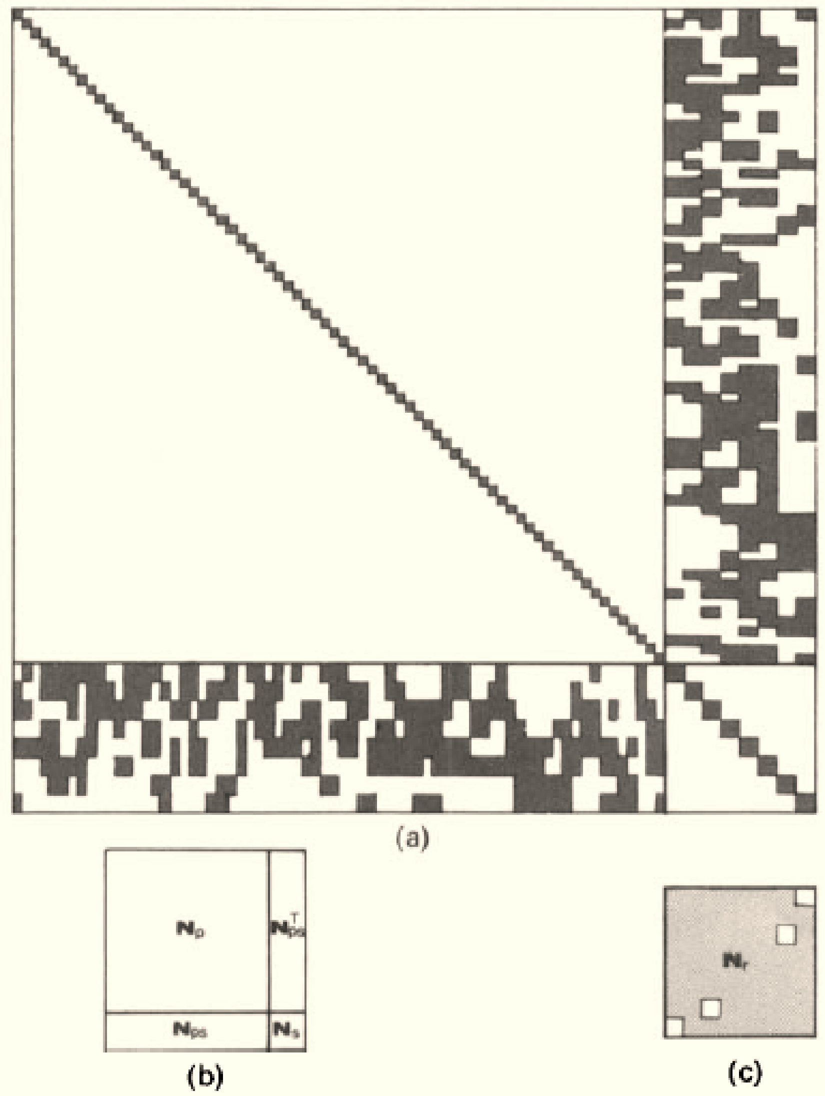


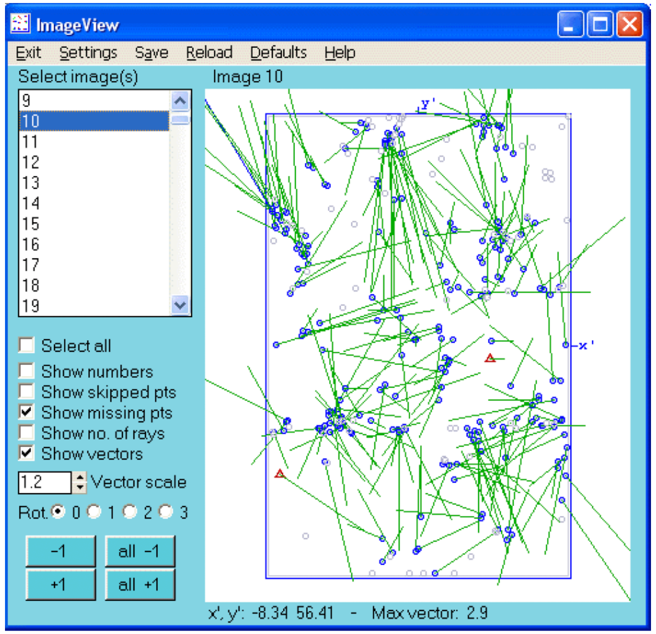
FIG. 3. (a) Structure of the coefficient matrix of the normal equations for 70 object points and eight pictures; (b) notation for the partitioned matrix given in (a); (c) the structure of the corresponding coefficient matrix of the reduced normal equations. Zero elements are left blank.

Granshaw, 2006, 185

Computational aspects … 4

- Despite all the above optimization, the enormous systems continued to challenge computer capacity, at least until around the turn of the century
- Various methods were used to speed things up, e.g. Levenberg-Marquardt, solving the normal equations directly, not updating everything on every iteration, not inverting the coefficient matrix until the last iteration, etc.
- There are subtle aspects, such as the relationships between the initial variances and covariances
- The end result has been some very sophisticated software

Computational aspects … 5



- Inclusion of external camera-related data
  - In the early days, statoscope, airborne profile recorder, edges of lakes
  - Now, GNSS/IMU: these can be point estimates for each exposure station, or can be modeled, e.g. drift parameters
- Robust estimation, i.e. no longer necessary to examine residuals manually and eliminate blunders one at a time – automation and revised weighting do the job
- Variance components
- Internal and external reliability
- Sensitivity analysis
- Network design – overlaps, cross-strips, ground control points etc.
- Just as important as all the above – excellent graphics to show results!

Sensor models

- So far, our entire discussion has been based on frame sensor models, i.e. collinearity equations or the projective geometry formulation for square or rectangular images, e.g. those captured by a film aerial camera
- What about digital aerial cameras that consist of multiple medium-format frame cameras?
- What do we do about other sensors, e.g. the pushbroom, whiskbroom and other sensors deployed from space, or uncalibrated or poorly calibrated cameras such as those in cell phones?
- In many cases, extensive sets of additional parameters suffice to model the real world well enough to obtain an acceptable solution

Sensor models … 2

- Or a different sensor model can be used, for example based on rational functions (RPCs)
- Some software systems have a small number of sensor models; some, especially SOCET SET and SOCET GXP, have a very large number indeed
- While most of the software programs developed for bundle adjustment form their Jacobian matrices of partial derivatives based on closed-form expressions, i.e. trigonometrical partials of the collinearity equations, others, such as SOCET SET and SOCET GXP, use numerical partial derivatives, which is usually much slower

The great German triangulators




- The great German triangulators
  - PAT-B: Fritz Ackermann, Heinrich Ebner, Hermann Klein (Inpho/Trimble, K^2 Photogrammetry)
    - PAT-M43, PAT-M7, PAT-B = “the Stuttgart programs”
    - For example, implemented on HP 21MX minicomputer of C100 Planicomp 1977
  - BLUH: Karsten Jacobsen (Hannover)
  - BINGO: Erwin Kruck (Gesellschaft für Industriephotogrammetrie mbH)
    - Scientist position with Konecny in Hannover, starting 1979
  - ORIMA: Ludger Hinsken (Leica Geosystems)
    - Implemented on Kern/Leica/Leica Geosystems analytical stereoplotters
  - CAP: Hermann Klein and Rüdiger Kotowski (K^2 Photogrammetry)
  - MATCH-AT/inBLOCK: Peter Krzystek, Tobias Heuchel et al. (Inpho/Trimble)

Other triangulators

- ALBANY (Adjustment of a Large Block of Anything): George Erio
- ISSBA: Riyadh Munjy (CSU Fresno)
- AeroSys: Matt Stevens (AeroSys Consulting LLC)
- Australis: Clive Fraser (Photometrix/GSI)
- ORIENT: Helmut Kager (T U Wien)
- GIANT (General Integrated Analytical Triangulation Program): Atef Elassal and Roop Malhotra (NOAA)

BLUH

Programmsystem

BLUH

BündeLblockausgleichung
Universität Hannover

HANNOVER

10.93

Program System

B L U H

BundLe block adjustment
University of Hannover

HANNOVER

10.93

Where does Karsten fit in?

- Devoted his life to bundle adjustment: PhD, Hannover, 1980: “Vorschläge zur Konzeption und zur Bearbeitung von Bündelblockausgleichungen”
- Development, support and proselytization of BLUH, the Hannover program for bundle adjustment
- Extensive work on integration of GNSS/IMU data
  - Lever arm and boresight calibration
- Continual improvement of the additional parameters, to accommodate imagery acquired from space as well as the new aerial cameras that consisted of multiple sensors, e.g. Z)I DMC and Vexcel UltraCam (additional parameters 81-88)
- Application of BLUH to examine the accuracy that could be obtained from all sorts of sensors, both airborne and spaceborne
- Superb presentations of his results
  - PowerPoints and papers rich with small tables, graphics and images to complement the text
- Involving numerous talented co-authors

Stuff in my library that mentions BLUH

- When I retired, I brought home 160 boxes of assorted “stuff”, including large numbers of technical papers and brochures; I scanned them all and saved them as searchable PDFs (>30,000 items)
- I looked in them for mentions of BLUH
- All the main suppliers mentioned BLUH in their brochures and often in their pricelists: Carl Zeiss Oberkochen, Carl Zeiss Jena, Inpho, Intergraph, Kern, KLT, Yzerman, Z)I etc.
  - Everybody interfaced to it!
  - Some vendors re-sold it

Stuff in my library that mentions BLUH ... 2

- Excerpt from Kern software description May 1982

4. CAPACITY

MAXIMUM NUMBER OF TERRESTRIAL POINTS	1800
MAXIMUM NUMBER OF PHOTOS	300
MAXIMUM NUMBER OF PHOTOS PER POINT	9
MAXIMUM NUMBER OF POINTS PER MODEL	500
MAXIMUM NUMBER OF POINTS TO COMPUTE OUTSIDE OF TRANSFORMATION	1000

COMPUTER:

CPU	11/23 OR LARGER
MEMORY	128KB MINIMUM
STORAGE	5 MB MINIMUM

Stuff in my library that mentions BLUH … 3

- Konecny, G., 1999, “Die Anfange der Photogrammetrie in Hannover”, Vortrag zum 50. Jubilaum des Instituts für Photogrammetrie und Ingenieurvermessungen der Universitat Hannover, 1 October 1999

> Das Bündelblockausgleichungsprogramm BLUH ist durch die kontinuierliche Forschungs- und Entwicklungstätigkeit von K. Jacobsen zu einem Markenbegriff geworden. Es läuft mit über 100 Installationen in allen Kontinenten und hat dem Institut eine solide finanzielle Basis geschaffen.

Stuff in my library that mentions BLUH … 4

- Papers from satisfied users
  - Kampsax India Private Limited (KIL)
  - Survey Department, Ministry of Transport, Public Works and WaterManagement, Delft, the Netherlands (Rijkswaterstaat Meetkundige Dienst.)
  - SwissPhoto
    - Many papers co-authored by Thomas Kersten, who will present the next keynote!

Karsten’s assessment

- "Today a block should be computed by a bundle block adjustment with self calibration by additional parameters. The bundle block adjustment is the most rigorous and flexible method of block adjustment. The computation with self calibration by additional parameters leads to the most accurate results of any type of block adjustment. Even based on the same photo coordinates an independent model block adjustment cannot reach the same quality; this is due to the data reduction by relative orientation, the comparatively inexact handling of systematic image errors and the usual separate computation of the horizontal and the vertical unknowns. In addition the photo orientations are required for several purposes and the other methods are not delivering this information.” (Jacobsen, 1999, 25)

Karsten’s assessment … 2

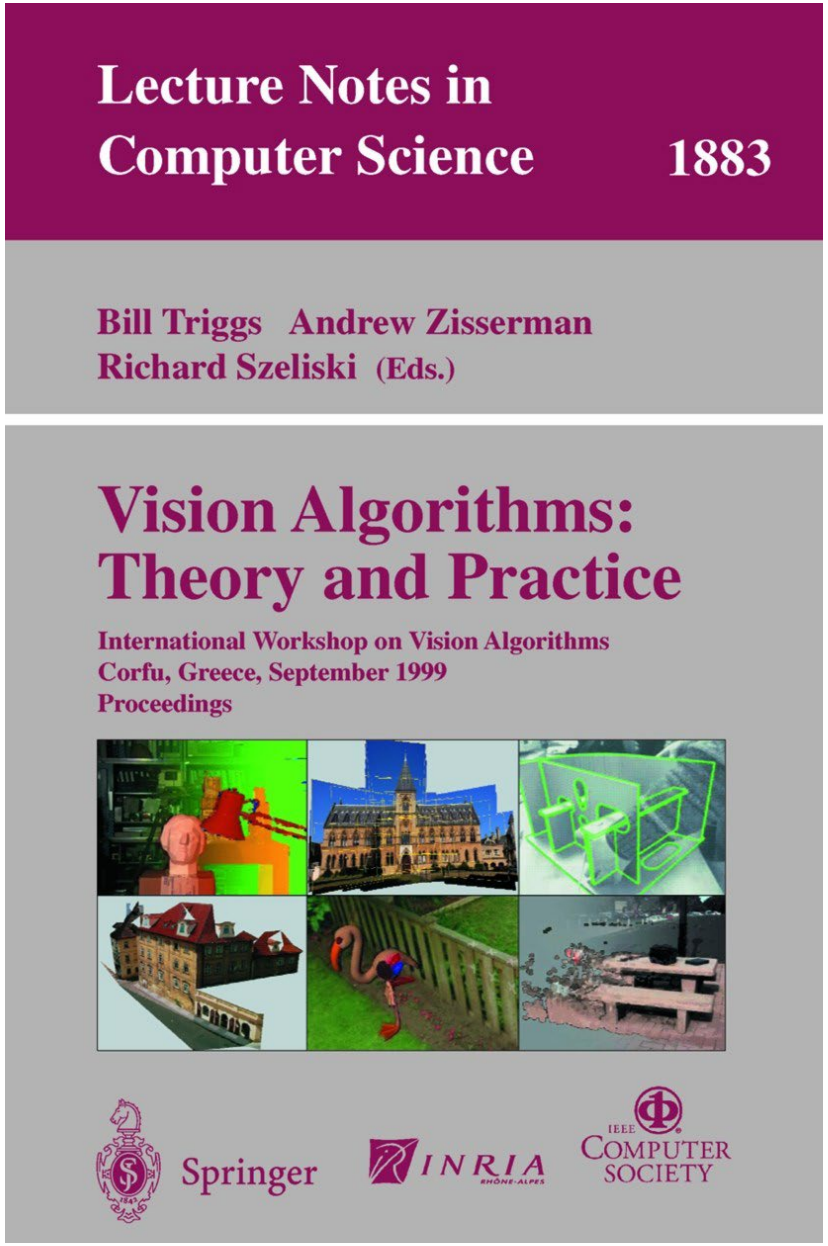
- "The bundle block adjustment is a very powerful tool, but good results only can be achieved if the input data are without problems and if a qualified program will be used. There are several bundle block adjustment programs on the market which are not qualified – by the operator support, by the support of the automatic blunder detection, by the self calibration and by the possibility of an analysis of the achieved results. In addition some programs do need a computation time in the range of hours while this can be done also in few seconds.” (Jacobsen, 1999, 25)

What happened next?

- We have sketched above the progress of traditional photogrammetric bundle adjustment – a gilded age indeed
- Computer vision people were working on it too - different nomenclature and different mathematics, but basically the same thing

Triggs et al. survey

- 75-page, detailed survey of bundle adjustment
- Published in *Vision Algorithms: Theory and Practice, International Workshop on Vision Algorithms, Corfu, Greece, September 21-22, 1999, Proceedings*; publication in 2000



Triggs et al. survey … 2

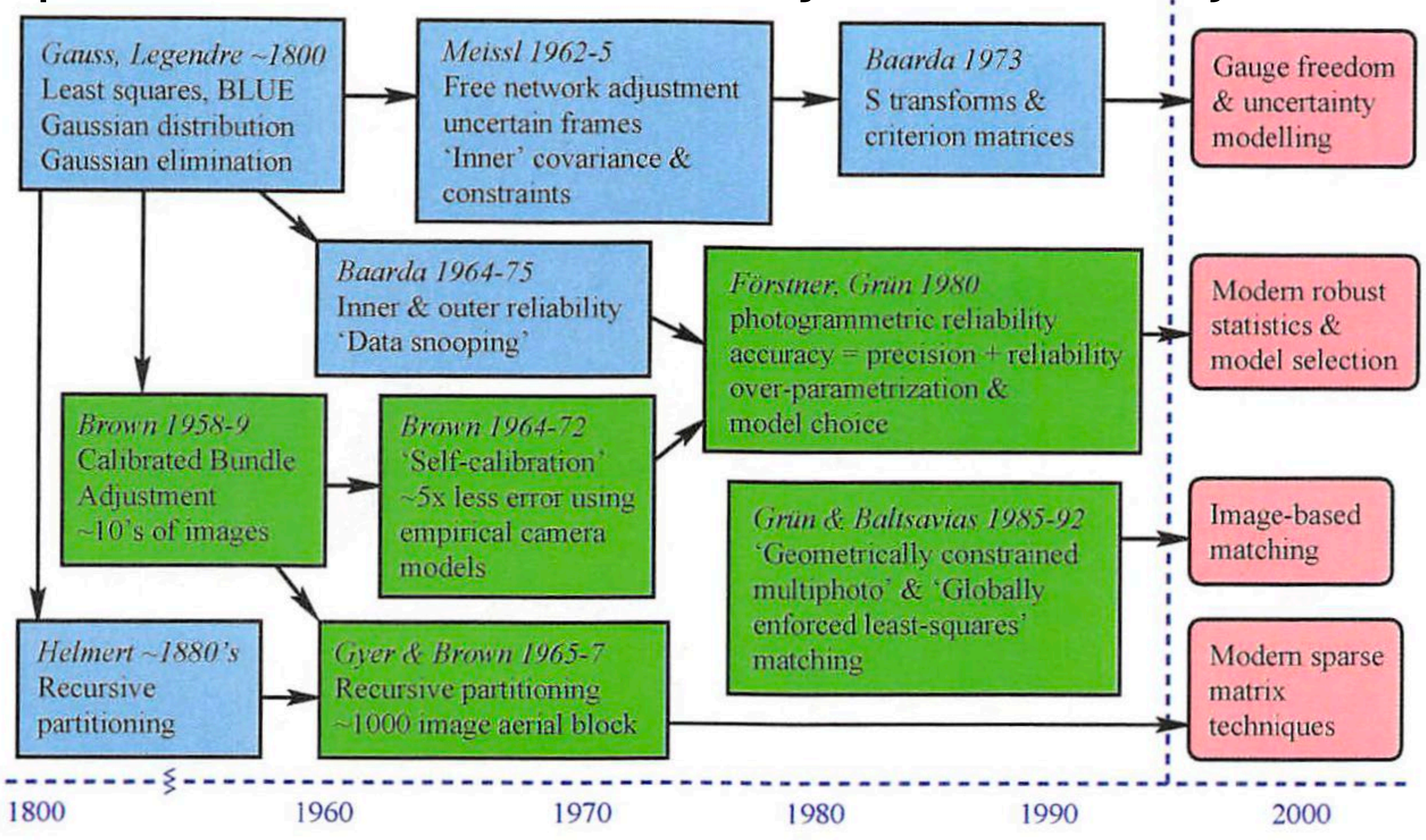
- Aimed at the computer vision community
- Still an extremely useful synthesis of the situation in 1999
- Great quotes:
  - “Most of the results appeared long ago in the photogrammetry and geodesy literatures, but many seem to be little known in vision, where they are gradually being reinvented.”
  - “Cultural differences sometimes make it difficult for vision workers to read the photogrammetry literature.”
- Acknowledgements to Zisserman, Grün and Förstner
- 37 of the 110 references are from photogrammetric literature, though around 10 are from surveying and geodesy

Triggs et al. survey … 3

- Computer vision has experimented with numerous cost functions, whereas the geospatial community has stuck to v^TWv
- Considerable attention to outliers and reliability in both communities
- "For this reason, we will not assume a least squares/quadratic cost model. Instead, the cost will be modelled as a sum of opaque contributions from the independent information sources (individual observations, prior distributions, overfitting penalties …)."
- Useful, 4-page history of bundle adjustment

Triggs et al. survey … 4

- Figure 9, p353, "A schematic history of bundle adjustment"



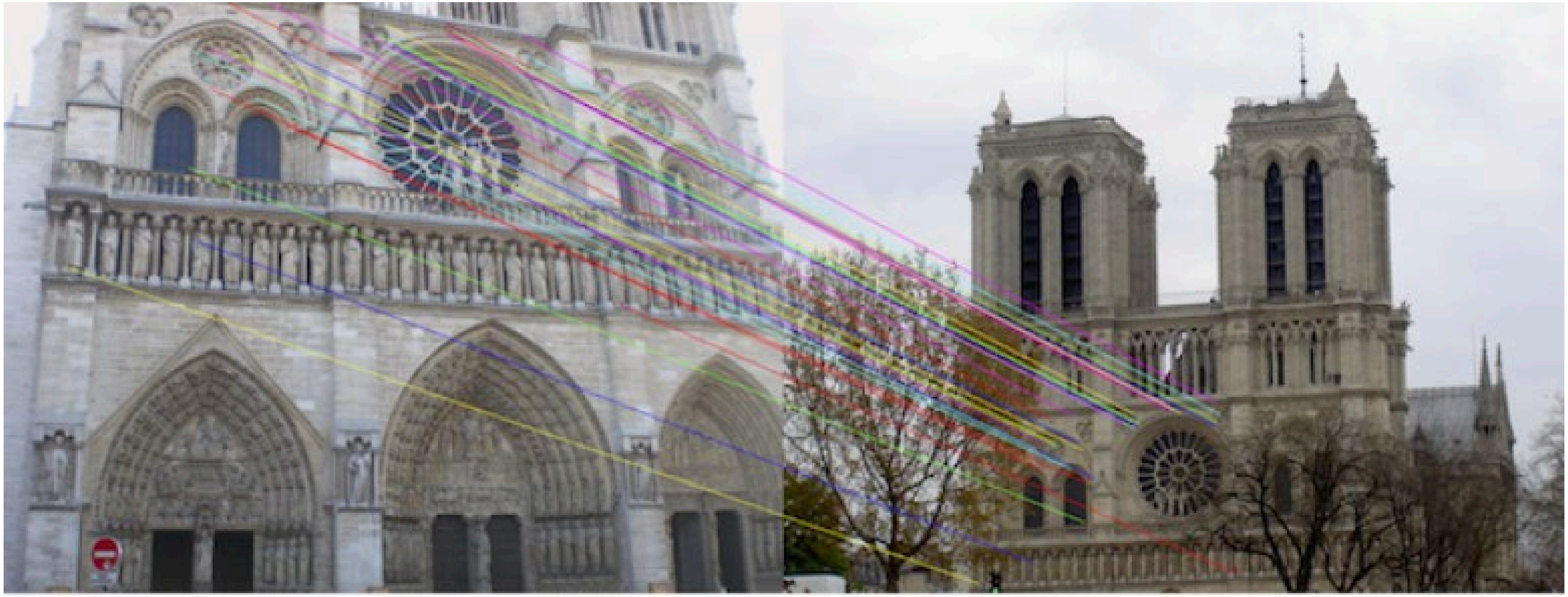
Triggs et al. survey … 7

- Parting quote:
  - “Quality control is a serious issue in measurement science, and it is perhaps here that the philosophical differences between photogrammetrists and vision workers are greatest: the photogrammetrist insists on good equipment, careful project planning, exploitation of prior knowledge and thorough error analyses, while the vision researcher advocates a more casual, flexible ‘point-and-shoot’ approach with minimal prior assumptions. Many applications demand a judicious compromise between these virtues.”

Important developments in 21st century

- Imagery is now digital (scanning aerial film was quite common in the 1990s and early 2000s, but has died out as digital cameras have replaced film)
- UAVs came along and UAV-photogrammetry brought new challenges:
  - Used by non-photogrammetrists
  - Elevation change in the scene can be a large proportion of flying height
  - Uncalibrated or poorly calibrated cameras
  - Low-cost GNSS/IMU systems providing initial approximations of variable quality
  - ”Front-ends” added to some packages to tackle problem of poor initial approximations

Important developments in 21st century … 2



Important developments in 21st century … 3

- Computer vision approaches to orientation of pairs of photos without the need for initial values
- Numerous new packages for bundle adjustment (free, open source, and for-profit; full packages or toolkits) with very high automation
  - Pix4D (various products) (Christoph Strecha et al.)
  - Agisoft (product currently called Metashape, formerly PhotoScan)
  - SBA (Sparse Bundle Adjustment)
  - COLMAP (Johannes Schönberger and Jan-Michael Frahm)
  - Bundler, VisualSFM, OpenMVG, Theia, GLOMAP and lots of others
- The term ”structure from motion” (SfM) now frequently used, often meaning bundle adjustment in the context of UAV-photogrammetry
- The large numbers of points output from the bundle adjustment are excellent for seeding the process of elevation extraction by dense image matching

Özyeşil *et al.* (2017)

- Talk about structure from motion, rather than bundle adjustment; but papers such as Özyeşil *et al.* (2017) and Schönberger and Frahm (2016) are informative successors of Triggs *et al*. (1999)
- SfM terms often encompasses feature description, feature matching, camera motion estimation and recovery of 3D structure; SLAM is a specific case (with lidar!)
- Focus on *unordered* image collections, but "…specific SfM algorithms targeting efficient solutions of relatively tightly constrained problem instances (e.g., accurate and fast depth estimation with known camera motion) will attract more attention in the future."

Özyeşil *et al.* (2017) … 2

- "…SfM community would still highly benefit from rigorous results on fundamental problems (e.g., what is the theoretically maximal amount of mismatched features or level of noise in the images that can be tolerated for a stable structure recovery, and can this be achieved efficiently?) and theoretical analysis of stability, robustness and computational efficiency of existing or new methods."

Schönberger and Frahm (2016)

- Incremental SfM
- COLMAP software
- Up to 100 million photos processed in one case, but, "While the existing systems have advanced the state of the art tremendously, robustness, accuracy, completeness, and scalability remain the key problems in incremental SfM that prevent its use as a general-purpose method." (Schönberger et al. 2017, 1)

Schönberger and Frahm (2016) … 2

- Bundle adjustment = tool/subset of the problems the CV community addresses, providing benefits:
  - Greater efficacy with large numbers of outliers
  - Approaches to initial approximations
  - Bringing in images sequentially
  - Focus on *unstructured* data sets – not the daily bread of photogrammetrists
- Problem image = WTF (watermarks, timestamps and frames)
- Exact and inexact step algorithms
- Scene graph augmentation
- Best next view
- Redundant view mining

Pan et al. (2019)

- Global SfM
- All two-view geometries in the scene graph considered simultaneously
- GLOMAP claimed to be faster, more accurate and more scalable than incremental SfM approaches such as COLMAP
- Rotation averaging then translation averaging; triangulation is merged with the latter

Current status

> “SfM addresses the problem of estimating the image poses and the corresponding sparse object 3D points. Over the past decade, SfM has obtained ample achievements, especially for large-scale image datasets, thanks to some popular open packages (Colmap, OpenMVG etc.). However, a challenging problem of matching visual overlapping image pairs is posed when dealing with very large image datasets, such as crowdsourced images of various landmarks and images collected from social media.”

Wang, X. and Y. Fen, 2024, BeBaOI: Benchmark and baseline methods for determining overlapping images, ISPRS SI2023 final report, 16 pp, 3.

Some current directions

- Bundle adjustment of images acquired with cameras containing rolling shutter
  - Yongcong Zhang et al., 2023, RSL-BA: Rolling shutter line bundle adjustment
  - Pix4D and others have done similar things
- Integration with deep-learning frameworks
  - Zitong Zhan et al., 2024, Bundle adjustment in the eager mode
  - "…we introduce an eager-mode BA framework seamlessly integrated with PyPose, providing PyTorch-compatible interfaces with high efficiency. Our approach includes GPU-accelerated, differentiable, and sparse operations designed for 2nd-order optimization, Lie group andLie algebra operations, and linear solvers."

Some current directions … 2

- Applications of advanced graph mathematics
  - Federica Arrigoni, Politechnico Milano, 2023. Viewing graph solvability in structure from motion, ISPRS Technical Commission II online talk, 20 September
    - Solvability of various systems – calibration helps
  - Andrea Fusiello, University of Udine, 2024. Graph synchronization and rigidity: unraveling the theory underneath structure from motion, ISPRS Technical Commission II symposium keynote 11 June
    - Establish consistent orientations for a set of sensors based on relative motion estimates, addressing the challenges of noisy edge measurements

Some current directions … 3

- Neural scene representation
  - Tianchen Deng et al, CVPR, 2024, 19657-19666, "PLGSLAM: Progressive neural scene representation with local to global bundle adjustment": neural scene representation – "The local-to-global bundle adjustment method combines the traditional SLAM method with end-to-end pose estimation, which achieves robust and accurate camera tracking and mitigate the influence of cumulative error and pose drift."
- Initialization-free bundle adjustment, i.e. without initial values
  - Simon Weber, Je Hyeong Hong, Daniel Cremers, 2024, "Power variable projection for initialization-free large-scale bundle adjustment"

Some current directions … 4

- Wavelets for self-calibration
  - Jun-Fu Ye, Jaan-Rong Tsay, and Dieter Fritsch, 2024, Wavelets for self-calibration of aerial metric camera systems, *Photogrammetric Engineering & Remote Sensing*, 90(9): 575-587

Endnote

- Bundle adjustment has developed remarkably since its inception, assisted partially by the evolution of photogrammetry as a whole through its analog, analytical and digital paradigms, but more through the booming of computer power and, more recently, the influence of the computer vision community
- From the early days of measuring the minimum practical number of image points necessary to join images together and orientate them, we can now process tens or hundreds of thousands of images automatically, with myriad points on every image
- Still a requirement for human understanding of the results and, occasionally, for humans to help the process along

Endnote … 2

- We honor the enormous contributions of Karsten Jacobsen to
  - Bundle adjustment through the development, support, proselytization and success of BLUH – a true leader during the first gilded age
  - The application of bundle adjustment for the analysis of imagery acquired from film and digital airborne cameras and from space, to assess its efficacy for various applications and investigate the performance of real-world systems, working productively and collegially with many students and assistants along the way
  - Meticulous, exact science, wonderfully presented and published
- And, throughout all that, he's remained a very nice guy!
- Thank you, Karsten. Congratulations on entering your 82nd year!